A tutorial on Fractons

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Outline

Introduction

• Condensed Matter Physics: Emergent phenomena in many-body systems.
Ex: quasi-particles, holes, Cooper pairs, anyons, ...,fractons!

• Quantum phases of matter $\Rightarrow$ emergence of exotic quasi-particles (fractional/non-Abelian statistics, fractional charge, etc);

• A new hype: Sub-dimensional quasi-particles with "fractional mobility";

• The topic has overlap with computer science, field theory and even gravity.

Some Recent History

The hype started around $\sim$ 2015, when people realized this was a new phase of matter (Vijay et al., 2015)

A New Kind of Topological Quantum Order:
A Dimensional Hierarchy of Quasiparticles Built from Stationary Excitations

Sagar Vijay, Jeongwan Haah, and Liang Fu
Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139

• "We term these fundamental excitations that behave as fractions of mobile particles 'fractons' ".(Vijay et al., 2015)

Before that, however, some models (containing fractons) were already known

- Chamon Code (Chamon, 2005)

Quantum glassiness in clean strongly correlated systems: an example of topological overprotection

Claudio Chamon
Physics Department, Boston University, Boston, MA 02215, USA

- Haah Code (Haah, 2011)

Local stabilizer codes in three dimensions without string logical operators

Jeongwan Haah*
Institute for Quantum Information, California Institute of Technology, Pasadena, CA 91125, USA
(Dated: 28 February 2011)

Fracton Systems

So far, we still lack a precise definition of what a fracton system is, but they usually share the common properties:

- Topological order (gapped);
- Restricted mobility of quasi-particles $\Rightarrow$ Sub-dimensional particles
  - $\sim$ 1-dimensional particles $\Rightarrow$ Lineons;
  - $\sim$ 2-dimensional particles $\Rightarrow$ Planons;
- Exotic Symmetries $\Leftrightarrow$ Unusual conservation laws;
- UV/IR scales mixing;

General Classification

There are two general classifications of fracton systems:

- Type-I

$\sim$ Isolated particles are immobile, but bound states can move. Ex: X-Cube model and Chamon code.

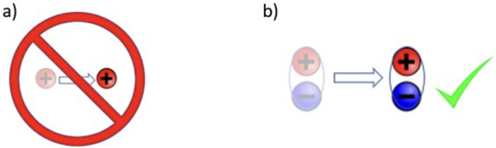


Figure: Figure borrowed from (Pretko, 2018).

- Type-II

$\sim$ All excitations are immobile. Ex: Haah Code

Motivations and Possibilities

- **CMT**: Non-trivial statistics in 3D dimensional systems;
  - $\sim$ Anyons in 3D;
- **CMT/CS**: Finite temperature quantum memories:
  - $\sim$ Large ground state degeneracy 2^L;
  - $\sim$ Particles struggle to hop around the system;
- **FT**: New advances in QFT :
  - $\sim$ UV/IR mixing;
  - $\sim$ Generalized symmetries;

Multiple approaches to fracton order:

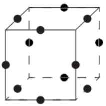
- Spin models on a lattice (Vijay et al., 2016);
- Continuum Lagrangians
  - $\sim$ Gapless higher-rank gauge theories (Pretko, 2018);
  - $\sim$ Topological-like gapped field theories (You et al., 2020);
  - $\sim$ Multipole algebra (Gromov, 2019);
- Stacks of coupled layers (Ma et al., 2017);
- Parton constructions (Hsieh and Halász, 2017);
- Generalized fusion theory (Pai and Hermele, 2019);
- and more...

X-Cube Model → Fracton Type-I

- There is a two-dimensional Hilbert space $\mathcal{H}_\ell$ (qubit) on every edge ℓ of the three-dimensional cubic lattice. The total Hilbert space is $\mathcal{H} = \prod_\ell \otimes \mathcal{H}_\ell$

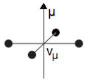
- We define the cube operator

$$B_{\mathcal{C}} \equiv \prod_{\ell \in \partial \mathcal{C}} X_\ell = \tag{1}$$



and star operators

$$A_v^\mu \equiv \prod_{\ell \in v_\mu} Z_\ell = \quad . \tag{2}$$



X-Cube model → Fracton Type-I

- We then define the interacting Hamiltonian among the spins to be

$$H_{XC} = -\sum_{\mathcal{C}} B_{\mathcal{C}} - \sum_{\mu=x,y,z} \sum_{v_\mu} A_v^\mu. \tag{3}$$

$\sim$ Exacly solvable;

- A ground state $|\psi_0\rangle$ belongs to the subspace

$$\mathcal{H}_0 = \{|\psi\rangle \in \mathcal{H} \text{ such that } B_{\mathcal{C}} |\psi\rangle = |\psi\rangle \text{ and } A_v^\mu |\psi\rangle = |\psi\rangle\} \tag{4}$$

$\sim$ Gapped $\Delta = 2$;

- Ground state is not unique. Large degeneracy in a periodic $L_x \times L_y \times L_z$ cubic lattice

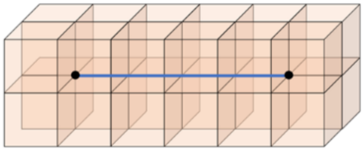
$$\log_2 \mathrm{GSD}_{\mathrm{XC}} = 2L_x + 2L_y + 2L_z - 3.$$

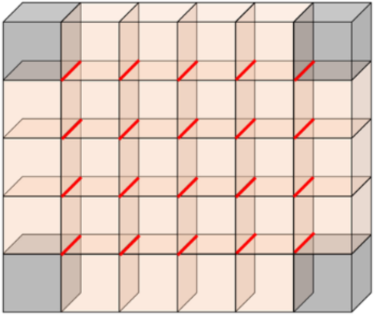
Properties of X-Cube

Excitations above the ground state:

- Lineons: violation of stars,happening at endpoints of rigid strings $\prod_{\ell\in\gamma} X_\ell$;
- Fractons: violation of cube operators, created at the corners of membrane operators $\prod_{\ell\in M} Z_\ell$;
- Lineons: Bound states of two fractons are one-dimensional particles;
- Local quasi-particles: Bound states of four fractons and dipoles of lineons;

$\Rightarrow$**Type-I fracton system!**

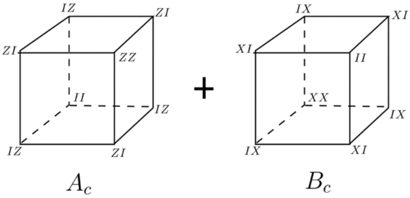




Haah Code → Type-II Fracton

- The Haah code consists of two qubits per lattice site with dynamics given by

$$H = -\sum_C \left(A_c + B_c \right) \tag{5}$$

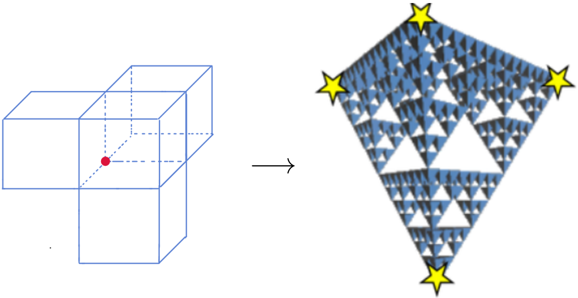


In a $L \times L \times L$ torus, GSD is a complicated non-monotonic function of L and is bounded by above

$$\log_2 GSD < 4L. \tag{6}$$

Haah Code Properties

- Excitations correspond to cube violations;



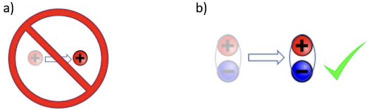
- 2.3219-dimensional particles;
- There are no string operators, even for bound states;

$\Rightarrow$ **Type-II Fracton!**

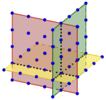
What about in the Continuum?

• As an attempt to describe particles with fractonic behavior, one can try to impose extra conservation laws:

∼ Higher multipole moment



∼ Subsystem symmetry



∼ and more...

Dipole Moment Conservation

- Fracton phenomenology $\Rightarrow$ generalized conservation laws

From usual Gauss Law, charge is locally conserved

$$\rho = \partial_i E_i \Rightarrow Q = \int_V d^3x \partial_i E_i = \oint_{\partial V} dn_j E_j \tag{7}$$

but not dipole moment

$$P_i = \int_V d^3x \, x_i \, \rho \neq \text{boundary integral.} \tag{8}$$

- Generalized Gauss Law (Pretko, 2018)

$$\rho = \partial_i \partial_j E_{ij}, \quad \text{with } E_{ij} \text{ a symmetric tensor} \tag{9}$$

$\Rightarrow$ **Scalar Tensor Gauge Theory!**

Generalized Gauss Law

The new Gauss law is the generator of generalized gauge transformations

$$A_{ij} \to A_{ij} + \partial_i \partial_j \alpha, \tag{10}$$

with commutation relation

$$[A_{ij}(\vec{x}), E_{kl}(\vec{y})] = i\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)\delta^3(\vec{x} - \vec{y}). \tag{11}$$

- Generalized Maxwell gauge theory

$$H \sim \int d^3x \left(\frac{1}{g_E^2} E_{ij}E_{ij} + \frac{1}{g_B^2} B_{ij}B_{ij} \right), \quad \text{with} \quad B_{ij} = \epsilon_{ikl}\partial_k A_{jl} \tag{12}$$

$\sim$ Gapless

- Higgs mechanism (Ma et al., 2018; Bulmash and Barkeshli, 2018b; Seiberg and Shao, 2021a);

Sub-System Symmetry

Exotic conservation equations might lead to huge number of conserved quantities;

• Example in 2d compact space (Seiberg and Shao, 2021b)

$$\partial_t \rho = \partial_x \partial_y J^{xy} \tag{13}$$

$\sim$ Besides the usual charge $Q = \int d^2x\, \rho$, it contains infinitely many extra charges

$$Q^x(x) = \oint dy \rho(x, y) \quad \text{and} \quad Q^y(y) = \oint dx \rho(x, y) \tag{14}$$

• In order to obey all these conserved quantities, charges configurations (as well as their mobility) are constrained;

$\Rightarrow$ **Subsystem Symmetry!**

Comments on Field Theories

• Tensor gauge theories relates to gravity, elasticity and Lifshitz (Pretko, 2017; Gorantla et al., 2022)

• These tensor theories usually present UV/IR mixing
  ∼ Ground state degeneracy depending on regularization;
  ∼ Discontinuous field configurations (Seiberg and Shao, 2021a)

• EFTs for both X-Cube and Chamon code (Slagle and Kim, 2017; Fontana et al., 2021);

• EFT for Haah (?) (Bulmash and Barkeshli, 2018a; Gromov, 2019; Fontana et al., 2022).

Final Remarks

$\Rightarrow$ Fracton phases is an intersection point of many fields: QFT, topological order, quantum information, non-ergodicity, gravity, ...

- Broad questions:
  - $\sim$ General classification of phases of matter?
  - $\sim$ Experimental realization?

- Canonical Reviews:

$\sim$ R. M. Nandkishore and M. Hermele, *Fractons*, Annu. Rev. Condens. Matter Phys. 10:295 - 313 **(2019)**

$\sim$ M. Pretko, X. Chen, Y. You, *Fracton Phases of Matter*, Intern. Journal of Modern Physics A Vol. 35, No. 06, 2030003 **(2020)**

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