Fusion Categories and SymTFT: Homework #10

Levin-Wen models

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Problem 1: Self-commuting projectors

Let A_v and B_p^s denote the vertices and plaquettes operators of the Levin-Wen model, respectively. Using the diagrammatic representation discussed in the lectures notes, first show that

$$B_p^{s_1} B_p^{s_2} = \sum_c N_{ab}^c B_p^c. \tag{1}$$

With this result at hand then prove that

$$[A_{v_1}, A_{v_2}] = 0 \ \ \forall \ \ v_1, v_2 \tag{2}$$

$$[B_{p_1}^{s_1}, B_{p_2}^{s_2}] = 0 \ \ \forall \ \ p_1, p_2, s_1, s_2. \tag{3}$$

Finally, show that for $a_s = d_s / \sum_k d_k^2$ the choice $B_p = \sum_s a_s B_p^s$ implies that

$$(B_p)^2 = B_p, \tag{4}$$

and therefore the Hamiltonian's ground state is made of a superposition of all possible states satisfying $A_v = B_p = +1$.

Problem 2: Chirality of modular fusion categories

The S and T matrix of the Fibonacci fusion category can be show to have the form

$$T_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & e^{4\pi i/5} \end{pmatrix}, \quad S_{ab} = \frac{1}{\sqrt{1+\varphi^2}} \begin{pmatrix} 1 & \varphi \\ \varphi & -1 \end{pmatrix}, \text{ with } \varphi = \frac{1+\sqrt{5}}{2}. \tag{5}$$

Given the identity of the modular group

$$(ST)^3 = e^{2\pi i c_-/8} S^2 \tag{6}$$

prove that the Fibonacci category has is Chiral, i.e. $c_- = 14/5 \mod 8$.