Fusion Categories and SymTFT: Homework #14

K-Matrix Formulation

May 6, 2026

Problem 1: Toric Code

Considering the Toric Code's K-matrix as

$$K = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix},$$

compute the filling fraction, the number of distinct quasiparticles, the ground state degeneracy on the torus, the superselection sectors, the charge of each particle, the exchange phase, and the statistics between each one of them.

Problem 2: Backscattering term

We have seen in class that the theory with K-matrix

$$K = \begin{bmatrix} 1 & 0 \\ 0 & -9 \end{bmatrix}$$

has 9 quasiparticles with superselection sectors given by

$$\mathcal{A} = \{(1,0),(1,1),...,(1,8)\}$$

and one Lagrangian subgroup

$$\mathcal{M} = \{(1,0),(1,3),(1,6)\}.$$

Haldane's condition $\Lambda^T K \Lambda = 0$ for this theory gives two possibilities of null vectors $\Lambda = (1, \pm 3)$. We have done in class the superconductor term with $\Lambda = (3, 1)$, where $m^T = (1, 3)$ is condensed at the edge, which in turn implies the condensation of all particles in the Lagrangian subgroup.

Now, analyze the backscattering term with null vector $\Lambda = (3, -1)$. What is the cosine term? The backscattering term will create/annihilate how many particles in each mode? What particle(s) is/are condensed? Does this correspond to the same Lagrangian subgroup as the superconductor term? Does this term conserve charge, or is the $U(1)$ symmetry broken?

Optional Homework: You can try a clever transformation on the fields as

$$\phi^2 = \varphi$$

$$\phi^1 + 3\phi^2 = \theta$$

and see in what minimum values the field is pinned. What symmetry emerges in this case?