

# Fusion Categories and SymTFT: Homework #8

Braided Categories

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[Adapted from Andrei Bernevig & Titus Neupert's 2015 lecture notes @ IHES]

## Problem 1: Topological Spin

Beyond its quantum dimension, each quasiparticle  $a$  in a braiding category is characterized by a unit-magnitude topological spin  $\theta_a$ . This value represents the phase gained during a  $2\pi$  self-rotation, equaling  $-1$  for fermions or a fractional phase for other anyonic species. The diagrammatic definition of  $\theta_a$  is

$$\text{Diagrammatic definition of } \theta_a \text{ showing two equations involving worldlines and circles.$$

Alternatively, we simply connect up the open worldlines with an  $\bar{a}$  worldline and obtain the definition

$$\theta_a := \frac{1}{d_a} \bar{a} \text{ (loop) } \bar{a} .$$

Show that this diagram can be evaluated by inserting an identity and applying an  $R$ -move and get

$$\theta_a := \frac{1}{d_a} \sum_c d_c \text{Tr} [R_c^{aa}] . \tag{1}$$

## Problem 2: S-matrix

In the TQFT, the  $S$  matrix is defined diagrammatically as

$$S_{ab} := \frac{1}{D} \text{Diagram of two overlapping circles labeled } b \text{ and } a \text{ with arrows pointing towards each other.}$$

where  $D$  is the total quantum dimension of the TQFT

$$D = \sqrt{\sum_a d_a^2} . \tag{2}$$

Show that, through diagrammatic manipulations, it can be written in terms of the fusion category data

$$S_{ab} = \frac{1}{D} \sum_c N_c^{ab} \text{Tr} \left( R_c^{ab} R_c^{ba} \right) d_c. \quad (3)$$