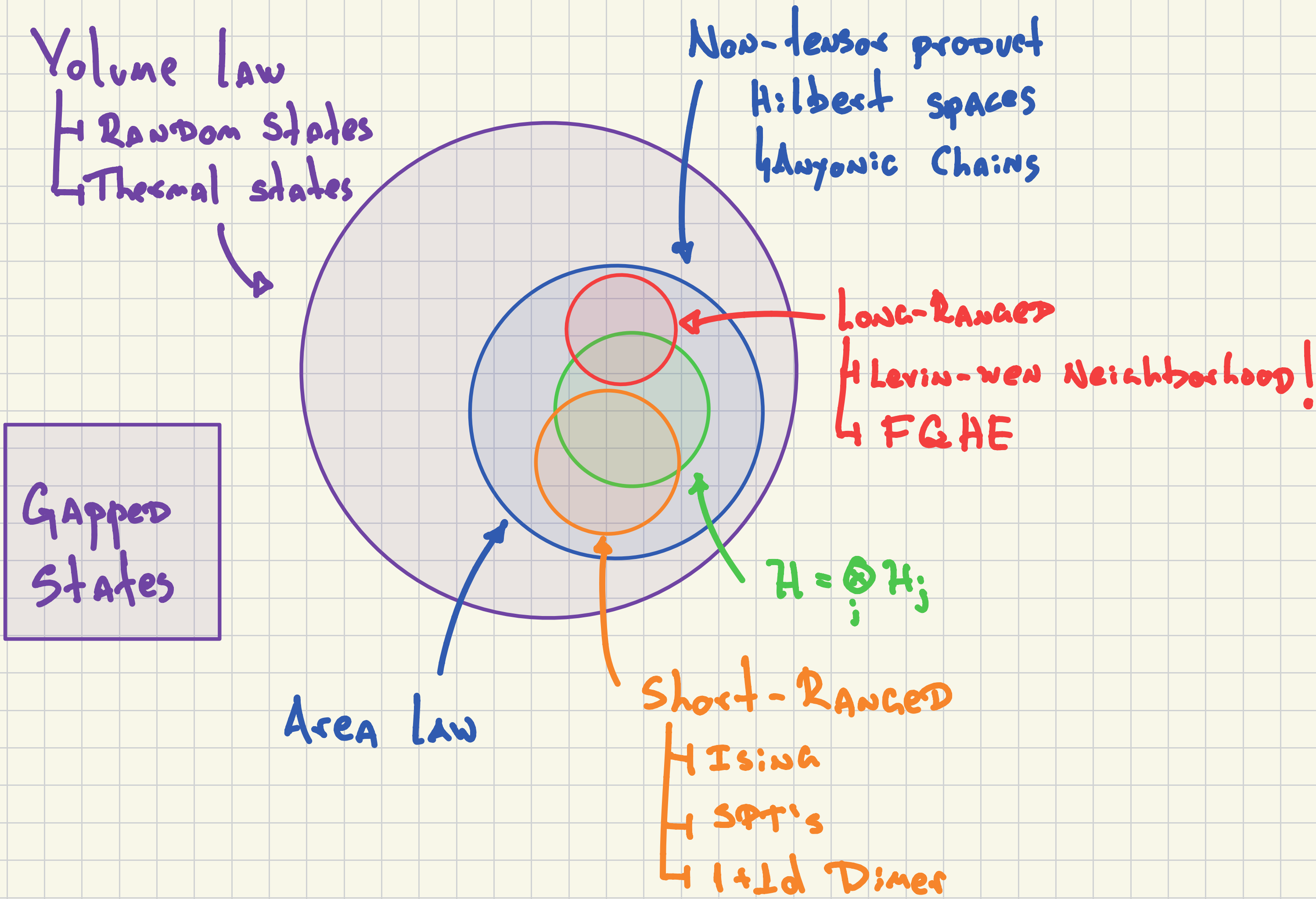
# String-Net Condensates

# Levin-Wen Models

What is the Nature of the model we will discuss?

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**STRING CONDENSATION—AN UNIFICATION OF LIGHT AND FERMIONS**

- **What are light and fermions?**
  Light is a fluctuation of nets of condensed strings of arbitrary sizes. Fermions are ends of condensed strings.
- **Where do light and fermions come from?**
  Light and fermions come from the collective motions of condensed string-nets that fill the space.
- **Why do light and fermions exist?**
  Light and fermions exist because our vacuum happens to have string-net condensation.

← Sacred text of topological orders!

Fusion Category $(d_a, N_{ab}^c, F^{ijk}_{lmn})$ $\longrightarrow$ Braided Fusion Categories $(d_a, N_{ab}^c, F^{ijk}_{lmn}, R_{ab})$

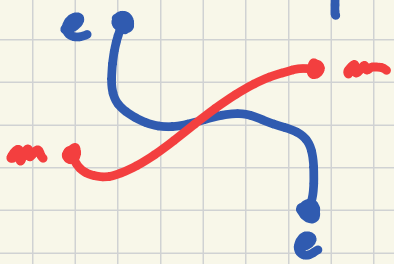
Levin-Wen models

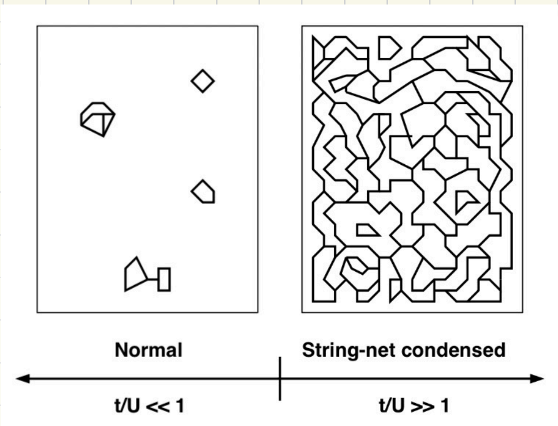
$\downarrow$

Anyons

→ Given $\mathcal{C}$ how to built a Hamiltonian? And study its particle spectrum!

$$H = -\sum_v A_v - \sum_p B_p$$



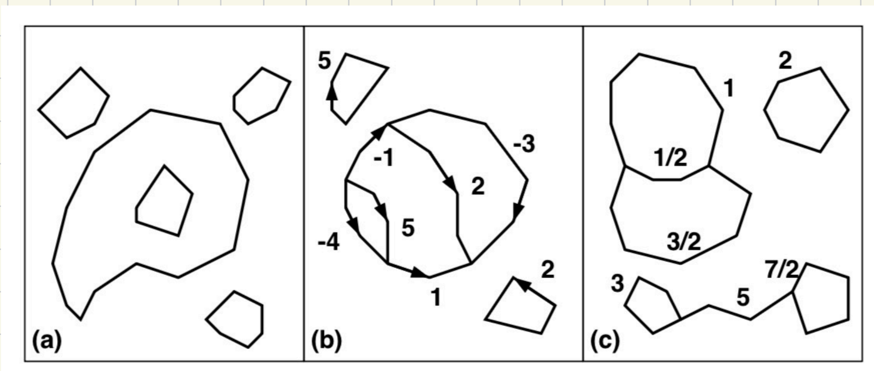




$$H = -t \sum_p \prod_{j \in p} \sigma^z - U \sum_i \sigma^x$$

Kinect Energy

String tension

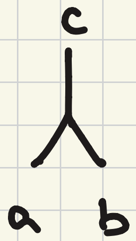
Branching rules:



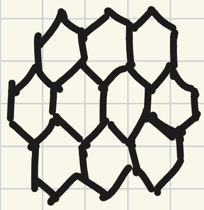

$\mathbb{Z}_2$ $\quad$ U(1) $\quad$ SU(2)

## The model:

→ We only consider trivalent 2D lattices



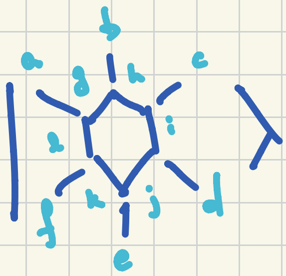
$\rightsquigarrow N_{ab}^{c}$



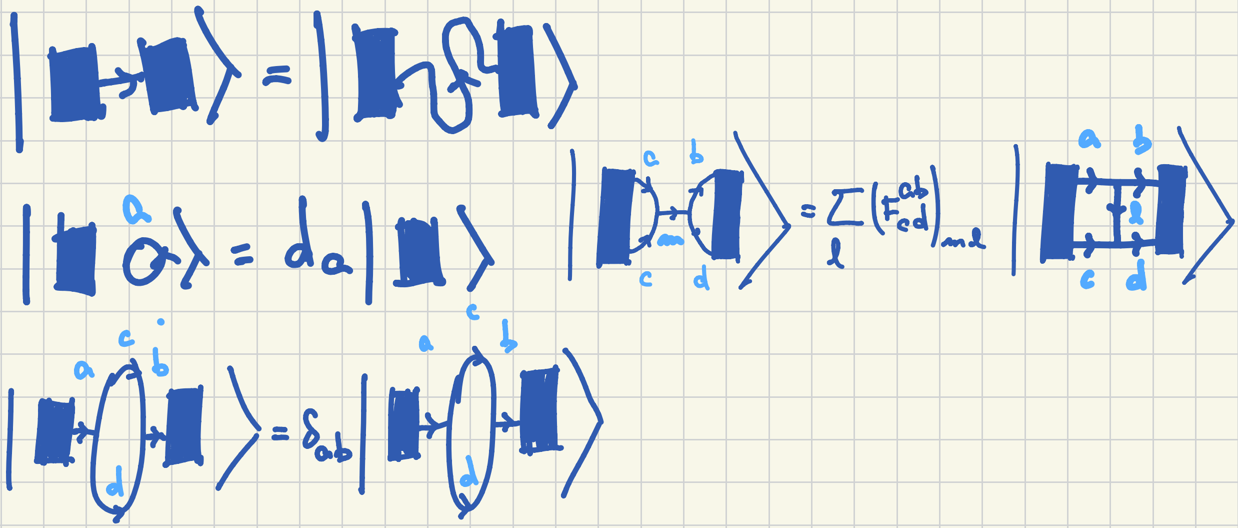
→ Degrees of freedom: Objects of the fusion category

↳ Hilbert is highly constrained by the allowed fusion (i.e. $N_{ab}^{c}$)

→ Diagramatic representation:



→ RG flow: We expect in the thermodynamical limit for such lattice description to flow to a fixed-point



We also require the fusion category to be unitary as well some self-consitency conditions

→ Pentagon equation

→ $$\left(F^{i j j^*}_{i^*}\right)_{k0} = \sqrt{\frac{d_k}{d_i d_j}}\, N_{ijk}$$

→ $$\left(F^{i^* j^* k^*}_{l^*}\right)_{a^* b^*} = \left(F^{ijk}_{l}\right)_{ab}$$

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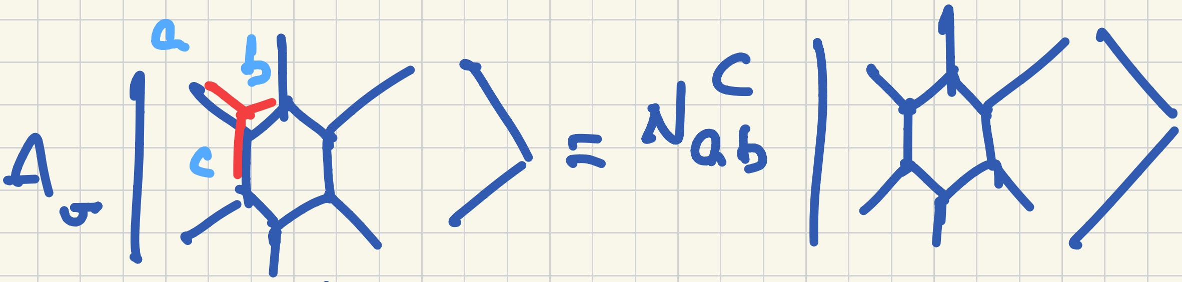
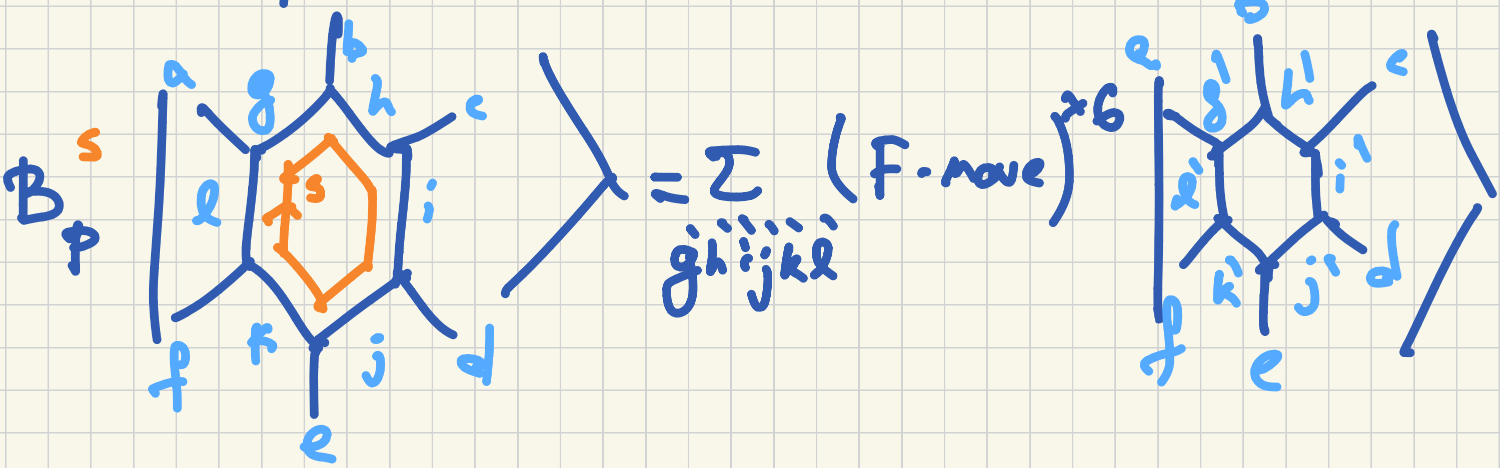
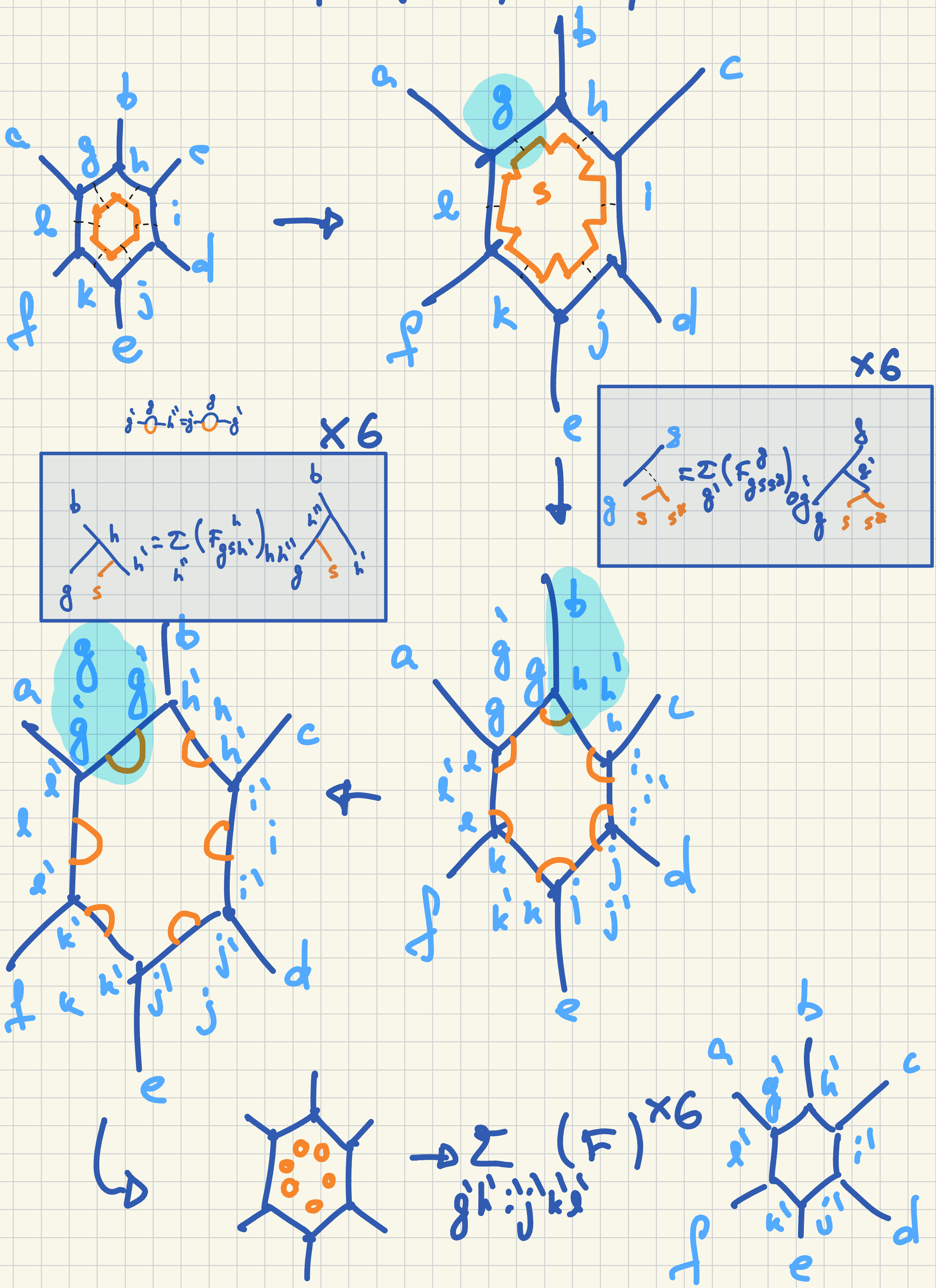
We define the plaquete and vertex operator as



# Sketch of proof $B_p^s$ :



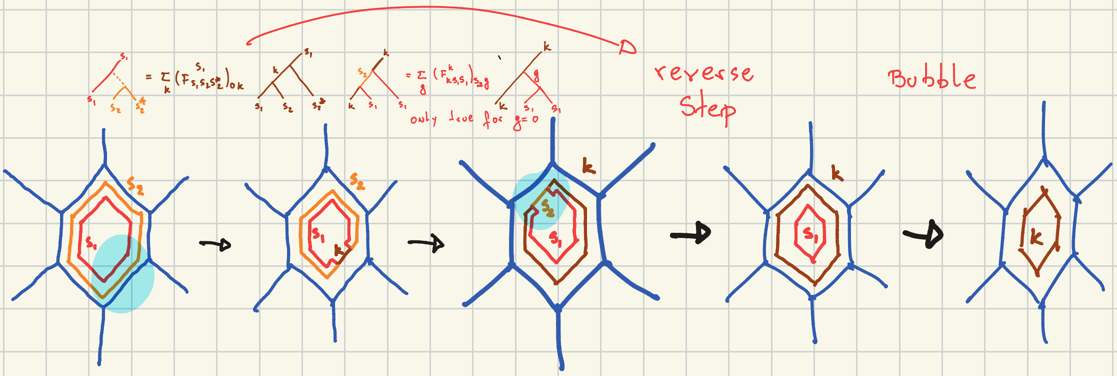
Homework: Prove

$$[A_\sigma, B_p^s] = 0 \quad \forall \sigma, p, s$$

$$[B_p^{s_1}, B_p^{s_2}] = 0 \quad \forall p, s_1, s_2$$

↳ For this first prove

$$B_p^{s_1} B_p^{s_2} = \sum_k N_{s_1 s_2}^k B_p^k$$



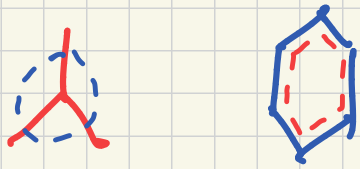

the Hamiltonian is then

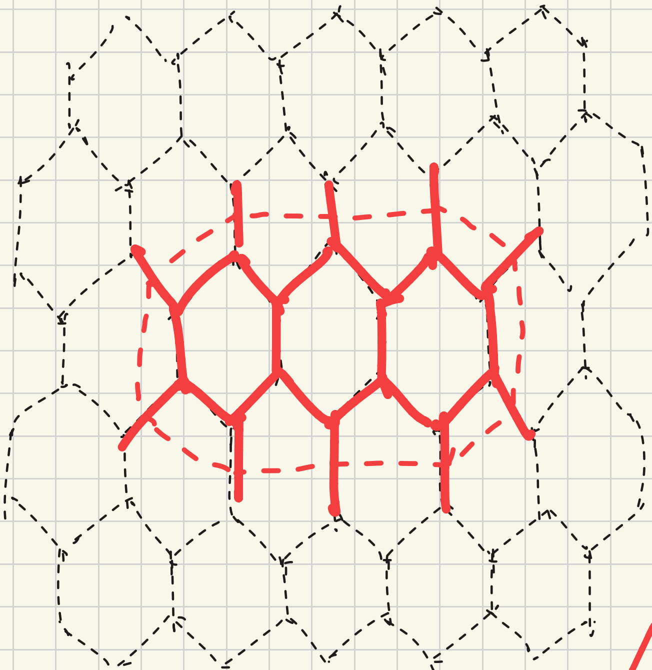
$$H = -\sum_\sigma A_\sigma - \sum_p B_p, \quad \text{with} \quad B_p = \sum_s a_s B_p^s$$

choosing $a_s = d_s / \sum_k d_k$ implies $B_p^2 = 1$

$$|GS\rangle \to A_\sigma = 1 \quad B_p = 1 \quad \forall \sigma, p$$

# line operators:





$$= \prod_{p \in C} B_p^s = W^s(C)$$

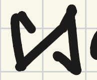
$$W^a W^b = \sum_c M^{ab}_c W^c$$

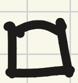
Homework !
Bruno

solutions satisfying $[H, W^a] = 0$
will classify the particle content of the theory

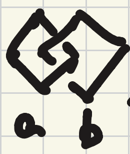
self-statistics | mutual-statistics

$e^{2\pi i s_a} = \dfrac{\langle \ \ a \rangle}{\langle \ \ a \rangle}$





$S_{ab} = \dfrac{1}{D} \langle \ \ \rangle$



a b

# Example: rank 2 categories

$\mathrm{obj}: \{\mathbb{1}, \alpha\}$ $\quad \alpha \otimes \alpha = N^{\mathbb{1}}_{\alpha\alpha}\, \mathbb{1} \oplus N^{\alpha}_{\alpha\alpha}\, \alpha$

$N^{1}_{\alpha\alpha} = 1 \quad N^{\alpha}_{\alpha\alpha} = 0$

$$\begin{aligned} d_0 &= 1 \\ d_1 &= F^{110}_{110} = \pm 1 \\ F^{000}_{000} &= F^{101}_{101} = F^{011}_{011} = 1 \\ F^{000}_{111} &= F^{110}_{001} = F^{101}_{010} = F^{011}_{100} = 1 \end{aligned}$$

## Toric Code

$$e^{i\theta_1} = 1,\ e^{i\theta_2} = 1,\ e^{i\theta_3} = 1,\ e^{i\theta_4} = -1$$

$$S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$\{1, e, m, f\}$

## Double semion

$$e^{i\theta_1} = 1,\ e^{i\theta_2} = i,\ e^{i\theta_3} = -i,\ e^{i\theta_4} = 1$$

$$S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$\{1, s, \bar{s}, b\}$

$N^{1}_{\alpha\alpha} = 1 \quad N^{\alpha}_{\alpha\alpha} = 1$

## Fibonacci $\left(\gamma = \frac{1+\sqrt{5}}{2}\right)$

$$F^{\alpha}_{\alpha\alpha\alpha} = \begin{pmatrix} \gamma^{-1} & \gamma^{-\frac{1}{2}} \\ \gamma^{-\frac{1}{2}} & -\gamma^{-1} \end{pmatrix}$$

$$e^{i\theta_1} = 1,\ e^{i\theta_2} = e^{-4\pi i/5},\ e^{i\theta_3} = e^{4\pi i/5},\ e^{i\theta_4} = 1$$

$$S = \frac{1}{1+\gamma^2}\begin{pmatrix} 1 & \gamma & \gamma & \gamma^2 \\ \gamma & -1 & \gamma^2 & -\gamma \\ \gamma & \gamma^2 & -1 & -\gamma \\ \gamma^2 & -\gamma & -\gamma & 1 \end{pmatrix}$$

$\{1, \tau, \bar{\tau}, b\}$

Note: Even though Fib is chiral

$$(ST)^3 = e^{2\pi i c_-/8} S^2$$

the resulting Anyon content is not

$\mathrm{Fib} \otimes \overline{\mathrm{Fib}} = \{1, \tau\} \otimes \{1, \bar{\tau}\}$