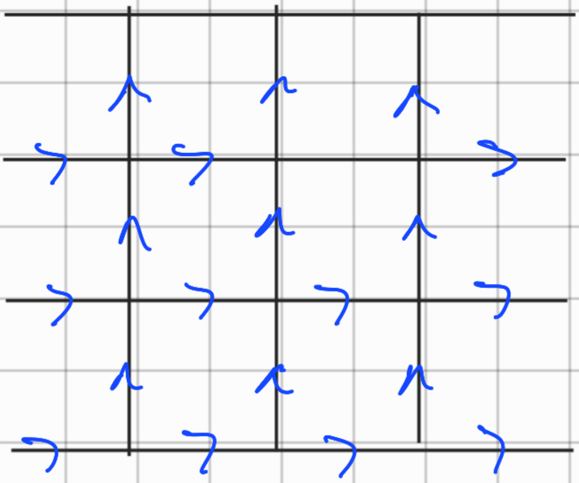
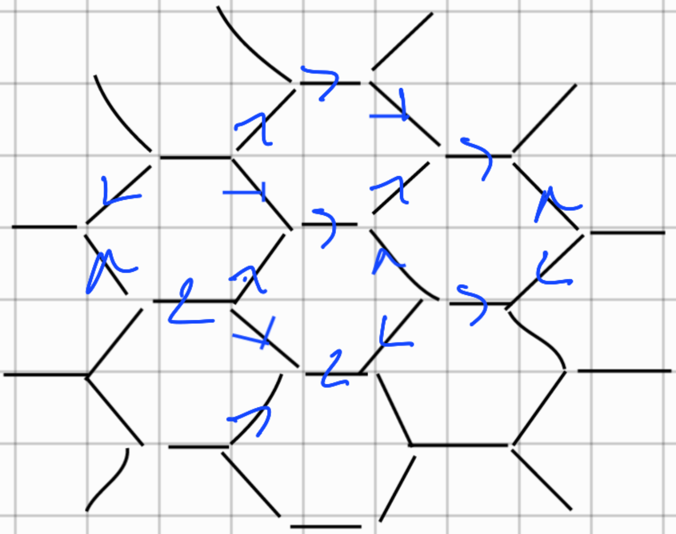
Quantum Double Model.

- G: finite Group
- (2+1) D lattice → planar.
  - → Oriented Graph



→ trivalent graph
→ square.

does not depend on the lattice geometry.



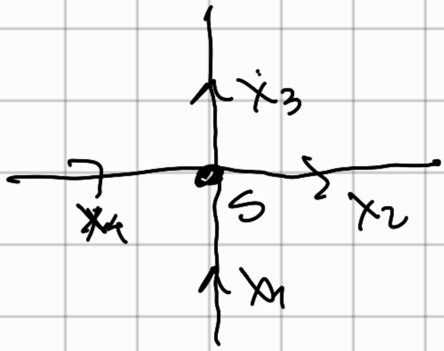
→ Orientation is a convention

- For each edge $e \in E$ hosts a Hilbert space $\mathcal{H}_i$, $\dim(\mathcal{H}_i) = |G|$

  Base: $|g\rangle$, $g \in G$.

- For each vertex

  $s \longrightarrow L_s$ a set of edges, i.e.,

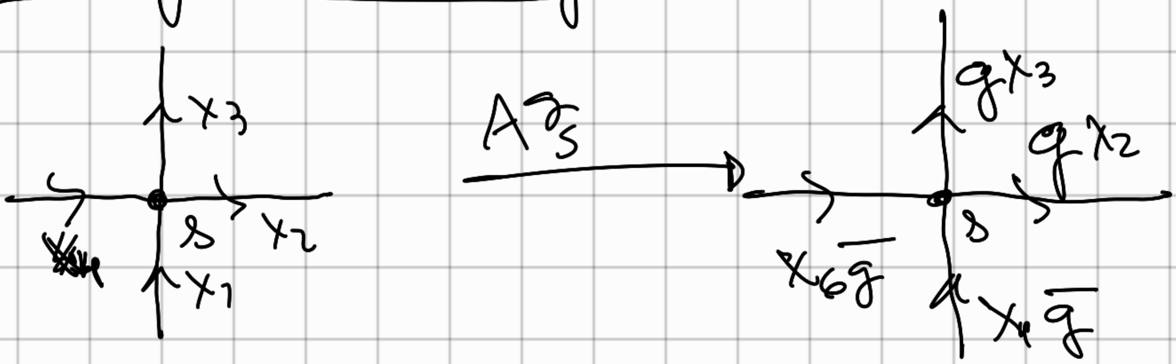


$L_s = \{x_1, x_2, x_3, x_4\}$

$x_i \in G$.

In this set we can act the star operator A^g_s, $g \in G$.

Diagramatically:



$$x_i \longrightarrow \begin{cases} g x_i & \text{if } x_i \text{ going out of } s \\ x_i \bar{g} & \text{if } x_i \text{ going into } s. \end{cases}$$

$$\bar{g} = g^{-1}.$$

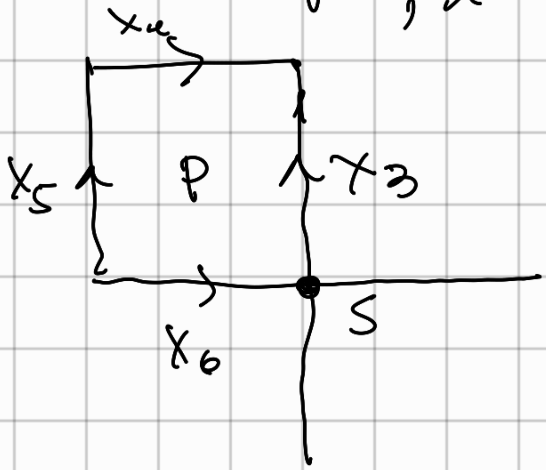
• <u>Notation</u>:

$$A^g_s |\underbrace{\ldots}_{\notin L_s}, x_1, x_2, x_3, x_6, \ldots\rangle =$$

$$= |\ldots, x_1\bar{g}, g x_2, g x_3, x_6\bar{g}, \ldots\rangle$$

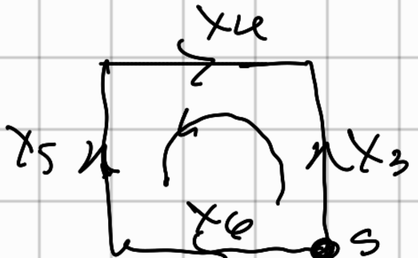
• $A^g_s \longrightarrow$ gauge transf.

• Given a face p, ∂p is a set of edges Γ_p, i.e.,

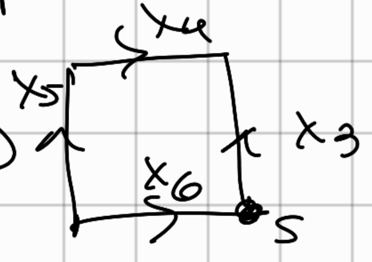


$$\Gamma_p = \{x_3, x_4, x_5, x_6\}$$

we can act the plaquette operator



$$\xrightarrow{B^g_p} \delta_{h, v(s,p)}$$



$U(s,p) = x_3 \bar{x}_4 \bar{x}_5 x_6,$

→ we start the concatenation in s in counterclockwise convention, and take the product $x_i (\bar{x}_i)$, x_i: aligned

$\bar{x}_i$: opposite.

<u>Notation</u>

$$B^h_{(s,p)} |\ldots, x_3, x_4, x_5, x_6, \ldots\rangle =$$

$$= \delta_{h, x_3 \bar{x}_4 \bar{x}_5 x_6} |\ldots, x_3, x_4, x_5, x_6, \ldots\rangle$$

$$= \delta_{h, U(s,p)} |\ldots\rangle$$

- B^h_p → project the holonomy (flux) on h.

The operators A^g_s, B^h_p satisfy:

$$(i)\quad A_s^{g_1} A_s^{g_2} = A_s^{g_1 g_2}$$

<u>Dem:</u> $A_s^{g_1}\left(A_s^{g_2}|x\rangle\right) =$

$$= A_s^{g_1}|\ldots, x_1\bar{g}_2, g_2 x_2, g_2 x_3, x_6 \bar{g}_2, \ldots\rangle$$

$$= |\ldots, x_1 \bar{g}_2 \bar{g}_1, g_1 g_2 x_2, g_1 g_2 x_3, x_6 \bar{g}_2 \bar{g}_1\rangle$$

$$= |\ldots, x_1 \overline{(g_1 g_2)}, g_1 g_2 x_2, g_1 g_2 x_3, x_6 \overline{(g_1 g_2)}\rangle$$

$$= A_s^{g_1 g_2}|x\rangle.$$

$$\boxed{\therefore\; A_s^{g_1} A_s^{g_2} = A_s^{g_1 g_2}.}\qquad (i)$$

$$(ii)\quad B_p^{h_1} B_p^{h_2} = \delta_{h_1, h_2} B_p^{h_1}.$$

$$\boxed{\begin{aligned} &\delta_{x,y}\,\delta_{z,y} \\ &= \begin{cases} 1, & x=y=z \\ 0 & \text{otherwise} \end{cases} \\ &= \delta_{xz}\,\delta_{xy}. \end{aligned}}$$

<u>Dem:</u>

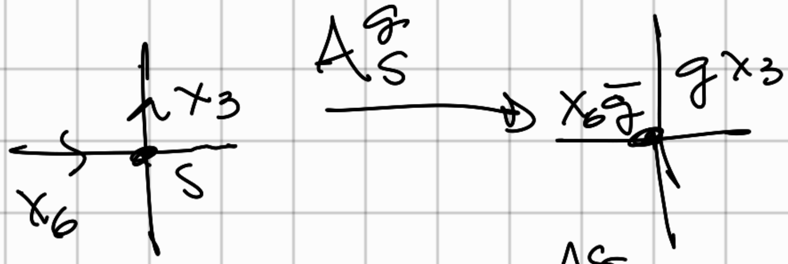
$$B_p^{h_1} B_p^{h_2}|x\rangle = B_p^{h_1} \delta_{h_2, \upsilon(s,p)}|x\rangle$$

$$= \delta_{h_1, \upsilon(s,p)}\, \delta_{h_2, \upsilon(s,p)}|x\rangle$$

$$= \delta_{h_1, h_2}\, \delta_{h_1, \upsilon(s,p)}|x\rangle = \delta_{h_1, h_2} B_p^{h_1}|x\rangle$$

(xiii) $A_s^g B_p^h = B_p^{ghg^{-1}} A_s^g$

<u>Dem:</u> (*) How $U(s,p)$ change under A_s^g.



$$U(s,p) \xrightarrow{A_s^g} g x_3 \cdots x_6 \bar{g}$$
$$= g\, U(s,p)\, \bar{g}.$$

<u>LHS:</u>

$$A_s^g B_p^h |x\rangle = A_s^g \delta_{h,U}|x\rangle = \delta_{h,U} A_s^g |x\rangle$$

<u>RHS:</u> $B_p^{gh\bar{g}} A_s^g |x\rangle =$

$$= B_p^{gh\bar{g}} |x'\rangle = \delta_{gh\bar{g},\, U'(s,p)} |x'\rangle$$

$$= \underbrace{\delta_{gh\bar{g},\, gU(s,p)\bar{g}}}_{\delta_{gh\bar{g},\, gU\bar{g}} = \delta_{h,U}} A_s^g |x\rangle$$

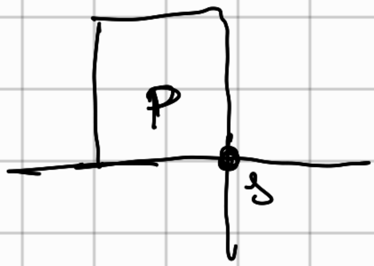
$$\Longrightarrow \delta_{h,U} A_s^g |x\rangle = \text{LHS}.$$

$$\boxed{\therefore A_s^g B_p^h = B_p^{gh\bar{g}} A_s^g.}$$

The operators A_s^g, B_p^h realize the $D(G)$ algebra: Drinfeld Double G (Hopf algebra)

- → Def: $D_{(g,h)} := A_s^g B_p^h$.

This is a algebra of local operator at site (s,p) on lattice.



- Ground state subspace

$V_{gs} = \text{span}_{\mathbb{C}}\{|\text{Vac}\rangle\}$ satisfying

$A_s^g|\text{Vac}\rangle = |\text{Vac}\rangle$,

$B_p^h|\text{Vac}\rangle = \delta_{h,e}|\text{Vac}\rangle$, $\forall s,p$.

↳ $e \in G$ (neutro)

A Hamiltonian with such ground state

$$H_G = -\sum_s A_s - \sum_p B_p^e \qquad e: \text{neutral}$$

where

$$A_s := \frac{1}{|G|} \sum_{g \in G} A^g_s .$$

A_s and B^e_p are commuting projectors.

<u>Dem</u> $A_s^2 = A_s$.

$$A_s^2 = \frac{1}{|G|^2} \sum_{g \in G} A^g_s \sum_{h \in G} A^h_s =$$

$$= \frac{1}{|G|^2} \sum_{g,h} A^g_s A^h_s = \frac{1}{|G|^2} \sum_{g,h} A^{gh}_s ,$$

<u>But</u>: $\sum_{g \in G} \sum_{h \in G} A^{gh}_s = |G| \sum_{k \in G} A^k_s .$

Aside: $|G| = 3$

same structure

$$\sum_{g,h} A_s^{gh} = \sum_{gh} gh \,.,$$

take a $g = g_1$ (fixed)

$g_1 \sum_h h$ is a ~~permeto~~ permutation of $|G|$ elements, i.e.,

$|G| = n$

$g_1(h_1, h_2, \ldots, h_n) = (h_1', h_2' \ldots, h_n')$ is a perm.

$g_2(h_1, h_2, \ldots, h_n) = (h_1'', h_2'', \ldots, h_n'')$ is ~~outher~~ other perm

$\vdots$

$$\left.\begin{array}{l} \\ \\ \\ g_n(h_1 \ldots, h_n) = (h_1^n, h_2^n, \ldots, h_n^n) \end{array}\right\} \text{total perm } |G| = n$$

then $gh = k$ ~~ope~~ on $|G|$ times

$$\Rightarrow \sum_{g,h} gh = |G| \sum_k k \,, \text{ such } k = gh.$$

$$\Rightarrow \sum_{g,h} A_s^{gh} = \sum_{k \in G} A_s^k \,.$$

Therefore:

$$A_s^2 = \frac{1}{|G|^2} \sum_{g \in G} \sum_{h \in G} A_s^{gh} =$$

$$= \frac{1}{|G|^2} |G| \sum_k A_s^k = A_s .$$

$$A_s^2 = A_s \quad \square$$

$$(B_p^h)^2 = B_p^h B_p^h = \delta_{h,h} B_p^h = B_p^h .$$

<u>A_s, B_p^e are projectors!</u>

$[A_s, B_p^e] = 0$, since

$$A_s B_p^e = \frac{1}{|G|} \sum_{g \in G} A_s^g B_p^e =$$

$$= \frac{1}{|G|} \sum_g B_p^{geg^{-1}} A_s^g =$$

$$= \frac{1}{|G|} \sum_g B_p^e A_s^g = B_p^e A_s .$$

$$\Rightarrow [A_s, B_p^e] = 0.$$

Since $[H_G, A_s] = [H_G, B_p^e] = 0$

The model is exactly solvable.

- Ground state

$A_s|Vac\rangle = |Vac\rangle$, $B_p^e|Vac\rangle = |Vac\rangle$

(sem carga) (sem fluxo)

eigenvalues of $A_s, B_p^e = \{0, 1\}$.

$$\Rightarrow \begin{cases} A_s|\psi\rangle = 0 & \text{(has an "electric" excitation on } s) \\ B_p^e|\psi\rangle = 0 & \text{(has an "magnetic" flux on } p) \end{cases}$$

Dimension of GS space (V_{gs})

$$\dim(V_{gs}) = |\mathrm{Hom}(\pi_1(F_i), G)/G|$$

$\mathrm{Hom}(X,Y)$: $\{\rho \mid \rho$ is a homomorphism of X in $Y\}$.

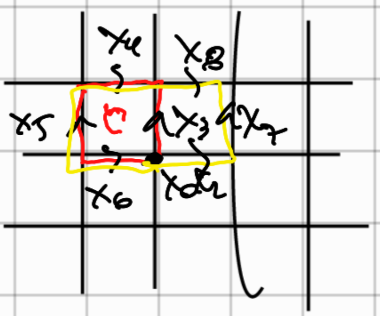
$\pi_1(F_i)$: fundamental group.

$\pi_1(F_i) = \{[\gamma],\ \gamma$ is a loop on $F_i\}$

$[\gamma] = \{\gamma'$ loop on F homotopic to $\gamma\}$.

• Intuição:

Dada uma variedade M e uma curva γ



$\gamma = e_3 e_4 e_5 e_6$, e_i vertices

$\gamma' = e_2 e_7 e_8 e_4 e_5 e_6 \notin G$.

$\mathrm{Hol}(\gamma) = X_3 \bar{X}_4 \bar{X}_5 X_6$

F (rede)

$\mathrm{Hol}(\gamma') = x_2 x_7 \bar{x}_8 \bar{x}_4 \bar{x}_5 x_6$

Reescrever $Hol(\gamma') =$

$$= X_2 X_7 \bar{X}_8 \bar{X}_3 X_3 \bar{X}_4 \bar{X}_5 X_6$$

$= Hol(\gamma_1)\, Hol(\gamma)$ if we restrict to $V_{gs} \Rightarrow Hol(\gamma_1) = e$.

Since we impose vacuum satisfy

$B_p^g |Vac\rangle = \delta_{g,e} |Vac\rangle \Rightarrow$

$Hol(\gamma)$ in a vacuum state $= e$.

Therefore $Hol(\gamma) = e$

$$Hol(\gamma') = Hol(\gamma_1)\, Hol(\gamma') = e \cdot e = e.$$

Then

$$Hol(\Gamma) \longrightarrow G$$
$$\gamma \longmapsto Hol(\gamma) \in G$$

does not depend of γ if the one homotopic

$\Rightarrow$ We can construct the

$$\rho : \pi_1(\Sigma) \longrightarrow G$$
$$[\gamma] \longmapsto \mathrm{Hol}(\gamma).$$

This is a homomorphism.

But we see ~~that~~ $\mathrm{Hol}(\gamma)$ is conjugated under G action:

And the quotient space is leading with this,

then we see ~~that~~ we need to take the quotient space of

$\mathrm{Hom}(\pi_1(\Sigma_1), G) / G$ $\to$ remove the gauge equivalent holonomy.

For G abelian $\mathrm{Hol}(\gamma) \to g\,\mathrm{Hol}(\gamma)\,g^{-1}$
$= \mathrm{Hol}(\gamma)$.

$$\Rightarrow \dim(V_{\Sigma}) = |\mathrm{Hom}(\pi_1(\Sigma_1), G)|$$

Example of Dim V_{gs} for Abelian G

- Consider $\Sigma = S^2$

$\pi_1(S^2) = \{e\}.$

$$\mathrm{Hom}(\pi_1(S^2), G) = 1$$

There is only one homomorphism,

$\rho([0]) = e_G$

homomorphism must map $e_H \to e_G$. $\forall H, G$.

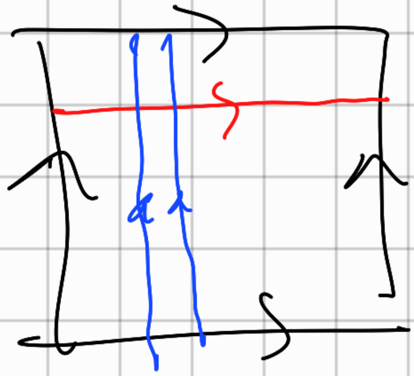
Therefore $\dim(V_{gs}) = |\mathrm{Hom}(\pi_1, G)|$
$= 1.$

no degeneracy!

<u>Ex</u> Take $\Sigma = T^2 = S^1 \times S^1$

$\pi_1(\Sigma) \cong \mathbb{Z} \times \mathbb{Z} = a^m b^n, \; m, n \in \mathbb{Z}.$

i.e., $\mathbb{Z} + \mathbb{Z}$ has two generators a, b independent



a: loop on x direction
b: loop on y direction

$\gamma = a b^2 = b^2 a.$

Define

$\rho : \pi_1(T^2) \longrightarrow G$

$a \longmapsto \rho(a) = g$

$b \longmapsto \rho(b) = h$

Since ab generate all elements in $\pi_1(T^2)$ then we need just to know how is ρ on a, b.

$$\rho(ab) = \rho(a)\rho(b) = gh = \rho(ba) = hg$$

$$\Rightarrow gh = hg.$$

So if G is abelian $\forall g, h$ satisfy this

$$\Rightarrow \text{Hom}(\pi_1(T^2), G) \cong G \times G \Rightarrow$$

$$\Rightarrow |\text{Hom}(\pi_1(T^2), G)| = |G \times G| = |G|^2$$

Therefore $\dim(V_{gs}) = |G|^2$,

<u>ex</u> $G = \mathbb{Z}_2$, $\dim(V_{gs}) = 2^2 = 4$

Anyon Content

Indexing irreducible representations (anyon content) of Drinfeld double
$G \qquad D(G)$

Hilbert space

$$\mathcal{H} = \bigoplus_{\alpha} \mathcal{L}_{\alpha} \otimes \mathcal{L}_{\bar{\alpha}},$$

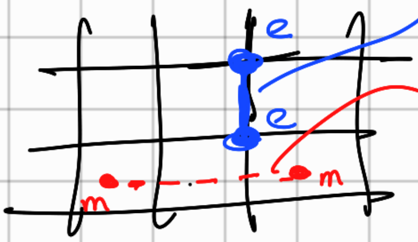
$\hookrightarrow$ antiparticle

α: anyon.

Decompose in superselection sectors.

The anyon is produce in same time with his antiparticle

→ Toric code:



line op. in original lattice.

line op in dual lattice

$m = \bar{m}$

$e = \bar{e}$

Each anyon type α of $D(G)$ is defined as following

(1) Conjugacy class of G

flux excitations

- Conjugacy class: Equivalence relation in a group G, given by:

$$a \sim b \iff \exists g \in G : b = gag^{-1}$$

Ex: Generic group G:

$\cdot$ $C(e) = \{ g \in G \mid g \sim e \}$

$$g \sim e \iff \exists h \in G :$$

$$g = heh^{-1} = e$$

$$\implies C(e) = \{e\} \ \forall G.$$

Ex: G is abelian:

$$C(g) = \{ h \in G \mid h = xgx^{-1} = \\ = g \}$$

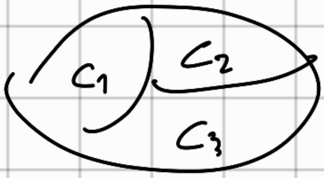
Therefore for G abelian

$$C(g) = \{g\} \quad \forall g \in G.$$

Aside:

Equivalence relation in a set $\longrightarrow$ Partition it in disjoint subsets

Ex: $G = S_3$



$C_i \cap C_j = \phi \quad i \neq j.$

(iii) An irrep of the centralizer representative member of the conjugacy class $C(g)$

$\longrightarrow$ "electric" charge

Aside: Centralizer of $g \in G$.

$$Z(g) = \{h \in G \mid hg = gh\}$$

$\longrightarrow$ is a subgroup of G

Ex: $\forall\, G,\ Z(e) = G.$

Ex: G abelian:

$$Z(g) = \{h \in G \mid gh = hg\} = G$$

Anyon $\longrightarrow (C(g), \rho)$

$C(g)$: conjugacy class

ρ : irrep$(Z(g') \mid g' \in C(g))$

- Consider G abelian
  - $C(g) = \{g\} \Rightarrow \# C(g) = |G|$

    therefore $\exists\ |G|$ type of flux.

  take a representative of $C(g)$

  there is only one g.

  $\Rightarrow Z(g) = G$

  For abelian G $\#\, \mathrm{irrep}(G) = |G|$

  Therefore $\exists\ |G|$ type of charge

So the $D(G)$ for abelian G has

$|G|^2$ type of anyons

$= \#\, GSD(\Sigma = T^2)$

• For any G:

$C(e) \longrightarrow Z(e) = G.$

$\Rightarrow$ # $\neg$ flux (no flux) anyon =
= # ireps(G)

$\forall\, C(g)$ we take $\quad$ irrep$\left(Z(g')\big|_{g' \in C(g)}\right) =$

= trivial

This correspond to a $\neg$ charge anyon

$\Rightarrow$ # conjugacy class =
= # $\neg$ charge anyon

But # conjugacy class =
= ireps(G) $\forall\, G$ (finite)

Therefore

$\neg$ flux anyons = # $\neg$ charge anyons

Ex: Consider $G = \mathbb{Z}_2 = \{1, m\}$

$|\mathbb{Z}_2|^2 = 4 \Rightarrow \exists$ 4 anyons.

$\#\text{ireps}(\mathbb{Z}_2) = 2$

$\Rightarrow$ 2 $\neg$ flux and 2 $\neg$ charge

but vacuum is $\neg$ charge and $\neg$ flux.

$\Rightarrow$ 1 vacuum, 1 $\neg$ charge, 1 $\neg$ flux

the other anyon has charge + flux.

vacuum $\longrightarrow 1$
$\neg$ charge $\longrightarrow m$
$\neg$ flux $\longrightarrow e$
charge + flux $\longrightarrow f$

Other way $D(\mathbb{Z}_2) = \{1, m\}$, $m^2 = 1$

$C(e) = \{e\} \longrightarrow Z(e) = \mathbb{Z}_2$
$C(m) = \{m\} \longrightarrow Z(m) = \mathbb{Z}_2$

Irreps $\mathbb{Z}_2$:

$$\chi_0(g) = e^{\frac{2\pi i \cdot 0 \cdot g}{2}} = 1 \quad \forall g$$

$$\chi_1(g) = e^{\pi i g}, \quad g \in \mathbb{Z}_2 = \{0, 1\}$$

χ_0 : trivial : $+$

χ_1 : sgn : $-$

Anyons:

$$(\{e\}, +) \to 1$$

$$(\{e\}, -) \to e$$

$$(\{m\}, +) \longrightarrow m$$

$$(\{m\}, -) \longrightarrow f.$$

Fusion:

- Abelian G:
  - $C(g) = \{g\}, \ \forall g \in G$
  - $Z(g) = G \Rightarrow \text{Irreps}(G), \forall g$.

anyon $\alpha = (g, \rho) \quad , \rho \in \{\text{Irreps}(G)\}$

$\beta = (g', \rho') \quad , \rho' \in \{\text{irreps}(G)\}$

$$\alpha \otimes \beta = (gg', \underbrace{\rho \otimes \rho'}_{\text{some group!}})$$

and ρ, ρ' are unidimentional.

$$\Rightarrow \alpha \otimes \beta = (\underbrace{g \circ g'}_{\text{group product}}, \underbrace{\rho\rho'}_{\text{usual multiplication in } \mathbb{C}})$$

<u>Ex:</u> $G = \mathbb{Z}_n = \{0, 1, \dots, n-1\}$,

$a, b \in \mathbb{Z}_n$,

$a \circ b = a + b \bmod n$.

irreps $(\mathbb{Z}_n)$,

$$\chi_k(g) = e^{\frac{2\pi i k g}{n}}, \quad k = 0, 1, \dots, n-1$$

$$(\chi_k \chi_p)(g) = (\chi_k \otimes \chi_p)(g)$$

$$= e^{\frac{2\pi i (k+p) g}{n}} = \chi_{k+p}$$

Therefore we can write the $D(\mathbb{Z}_n)$ fusion rule by:

$$\alpha = (g, \chi_k) = (g, k) \quad , g, k = 0, 1, \dots, n-1$$

$$\beta = (g', \chi_{k'}) = (g, k')$$

And the fusion is given by:

$$\alpha \otimes \beta = (g, k) \otimes (g', k')$$
$$= (g + g' \bmod n, \; k + k' \bmod n)$$

Ex: $n = 2$ $\quad D(\mathbb{Z}_2) =$ Toric Code:

$(0, 0) \to 1$
$(0, 1) \to e$
$(1, 0) \to m$
$(1, 1) \to f$,

Aside:
of course for $G = \mathbb{Z}_n$
$\alpha \otimes \beta = \beta \otimes \alpha$
trivially!

$$e \otimes m = (0, 1) \otimes (1, 0) =$$
$$= (0 + 1, 1 + 0) = (1, 1) = f.$$

$$f \otimes f = (1, 1) \otimes (1, 1) = (2 \bmod 2, 2 \bmod 2)$$
$$= (0, 0) = 1$$

Then for $\alpha \in D(\mathbb{Z}_n)$,

$\alpha = (g, k)$, $\bar{\alpha} = (g', k')$ satisfy:

$g + g' \bmod n = 0$

$k + k' \bmod n = 0$.

Quantum dimension

the quantum dimension is given by:

for a $\alpha \in D(G)$

$\alpha = (C(g), \rho)$

$d_\alpha = |C(g)| \dim(\rho)$.

Ex: Consider G abelian

then $C(g) = \{g\} \Rightarrow |C(g)| = 1$

$Z(g) = G$, $\rho \in \mathrm{Irrep}(G)$

$\Rightarrow \dim \rho = 1$. ($G$ is abelian)

Therefore $d_\alpha = 1 \cdot 1 = 1$, $\forall \alpha \in D(G)$ α is abelian.

• Now lets take a look for a non abelian G.

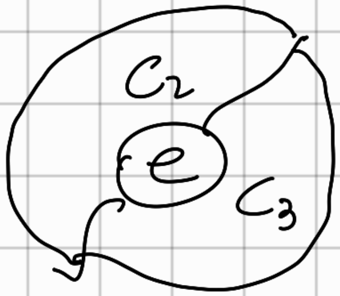
$G = S_3 = \{e, (12), (13), (23), (123), (132)\}$

Anyons Content

There are 3 conjugacy class:

$C(e)$, $C(12)$, $C(123)$

S_3 C_1, C_2, C_3



$\Rightarrow \exists$ 3 type of flux.

Consider $C_1 = \{e\} \Rightarrow Z(e) = S_3$

S_3 has 3 irreps, $+$ (trivial) $\to$ dim 1
$-$ (signal) $\to$ dim 1
(2) (std) $\to$ dim 2

Then we have 3 anyons with trivial flux

$(\{e\}, +) \to A = 1$ (vacuum), dim 1
$(\{e\}, -) \to B$ ($\neg$ flux), dim 1
$(\{e\}, [2]) \to C$ ($\neg$ flux), dim 2

Now consider $C_2 = \{(12),(13),(23)\}$

take (12) as representative

$Z(12) = \mathbb{Z}_2$, $\mathbb{Z}_2$ has # irreps $= 2$

Anyons

$D = (C(12), +) \longrightarrow$ (7 charge), dim 3

$E = (C(12), -) \longrightarrow$ Dyon , dim 3.

Consider $C_3 = C(123) = \{(123),(132)\}$

$Z(123) \equiv \mathbb{Z}_3$, # irreps $\mathbb{Z}_3 = 3$

$(1, \omega, \omega^* = \omega^2)$

Anyons

$\omega(g) = e^{\frac{2\pi i}{3} g}$

$\hookrightarrow g = \{0,1,2\}$

$F = (C(123), +) \rightarrow$ (7 charge)

$G = (C(123), \omega) \longrightarrow$ (dyon)

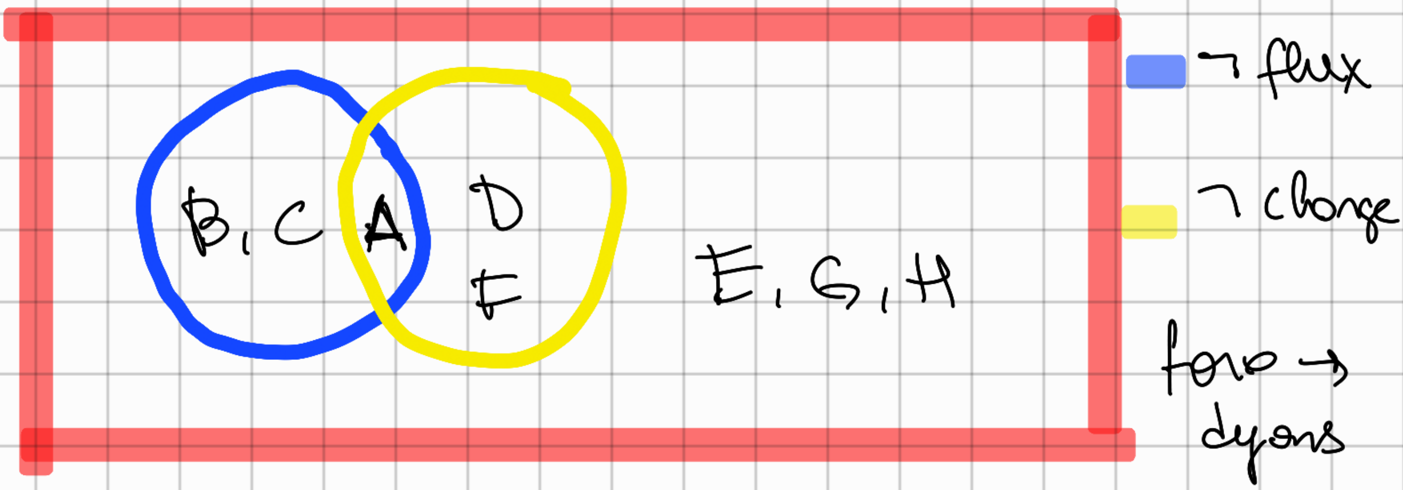
$H = (C(123), \omega^*) \longrightarrow$ (dyon)

$\Big\}$ dim 2

$\{A, B, C\} \longrightarrow$ trivial flux

$\{D, E\} \longrightarrow$ flux on class of odd perm.

$\{F, G, H\} \longrightarrow$ flux on class of even perm.



Aside: Maybe (Warning!)

$$\mathcal{L}_e = \{A, B, C\}$$

→ electric Lagrangian algebra

$$\mathcal{L}_m = \{A, D, F\}$$

→ magnetic Lagrangian algebra.

	C. class	Z	irrep	d	Label
	C_1	S_3	$+$	1	A
			$-$	1	B
			$[2]$	2	C
			$+$	3	D
$\lvert C_2\rvert=3$	C_2	$\mathbb{Z}_2$	$-$	3	E
			$+$	2	F
$\lvert C_3\rvert=2$	C_3	$\mathbb{Z}_3$	ω	2	G
			ω^*	2	H

non Abelian anyons.

<u>Fusion:</u> Take the fusion of one element is of trivial flux as easy! But with elements where the two flux are in class $(c(g) \neq c(e)$ we need caution!

Consider $\alpha = (c(e), \rho)$,

$\beta = (c(g), \rho')$

$$\alpha \otimes \beta = (c(e) \circ c(g), \rho \otimes \rho') =$$

$$= (c(g), \rho \otimes \rho')$$