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ENERGY
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Probing physics in arbitrary graphs and curved spaces

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arXiv: 2605.28942

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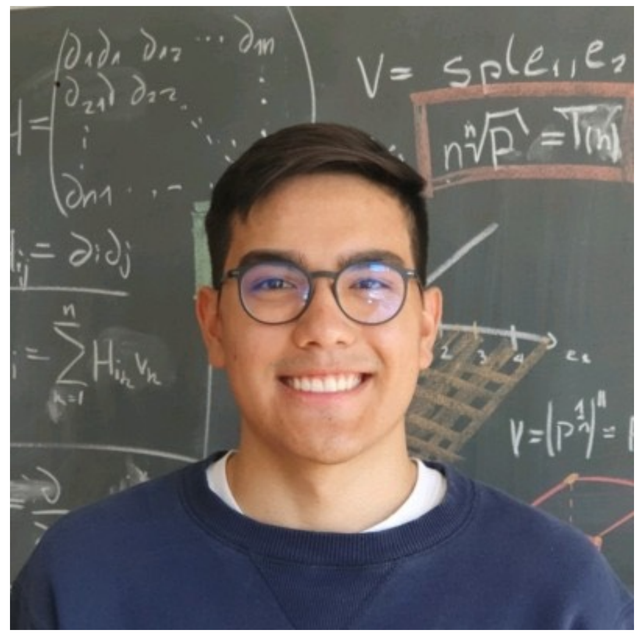

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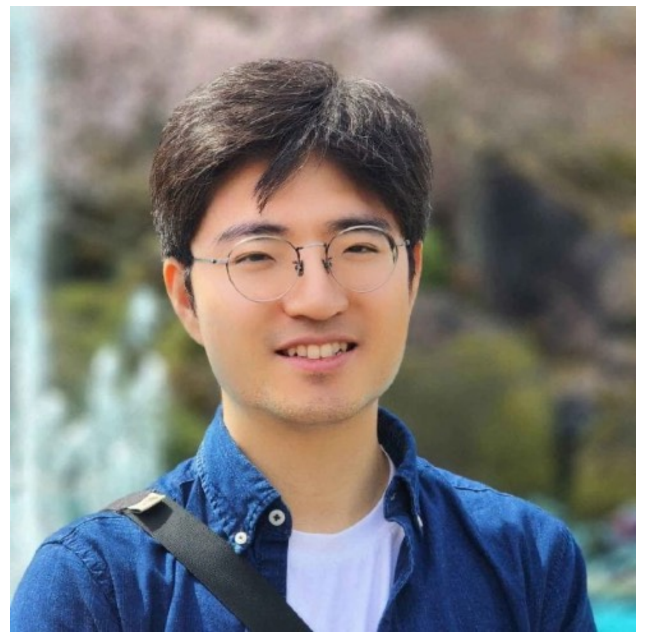

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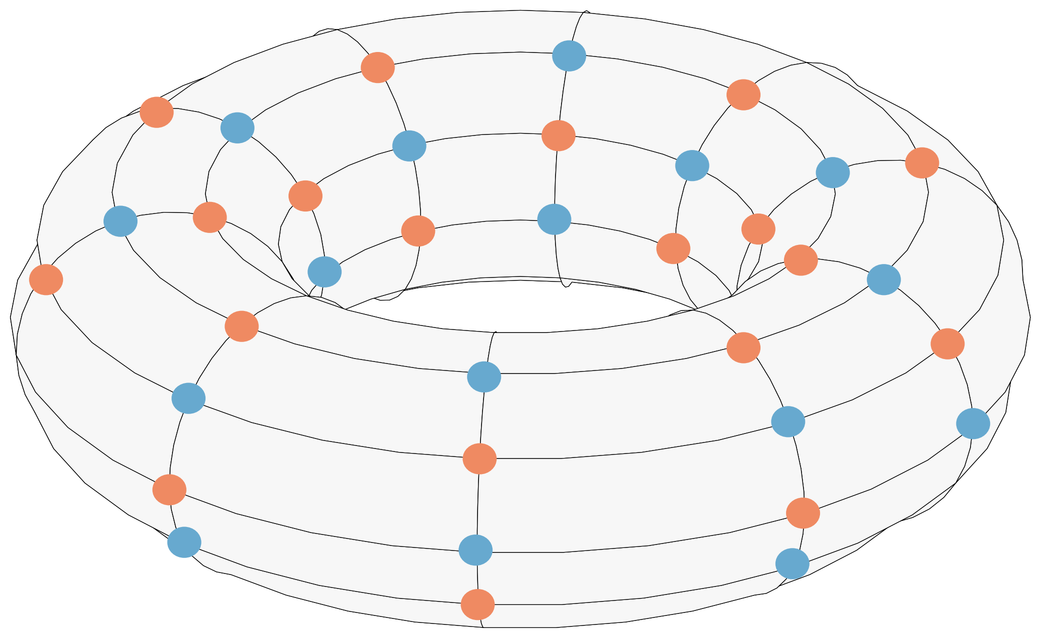
Probing Rich Physics

- Rich physics may emerge in/from:
  - Higher-dimensions (SSB, LRE, etc)
  - Non-trivial topologies (topologically ordered phases)
  - Curved spaces (AdS/CFT)



- Goal: experimentally probe higher dimensions and richer geometry/topology limitations
- Proposal: build such systems by using “non-local” degrees of freedom → Superconducting Wires

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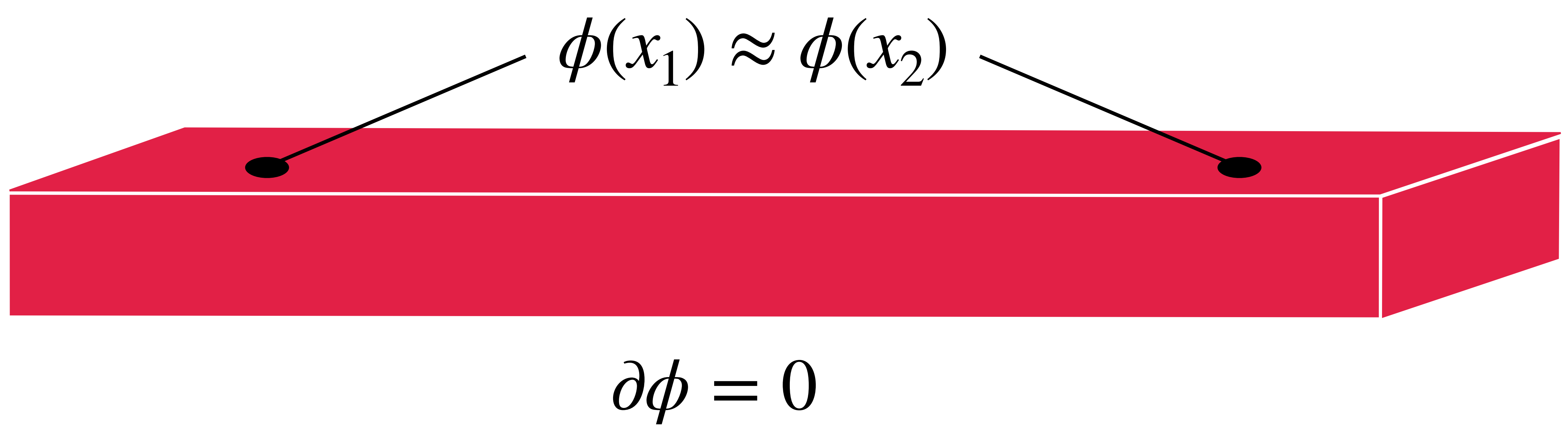
Superconducting wires

- Superconducting wires are characterized by phase ϕ, which behave as a single, rigid degree of freedom,

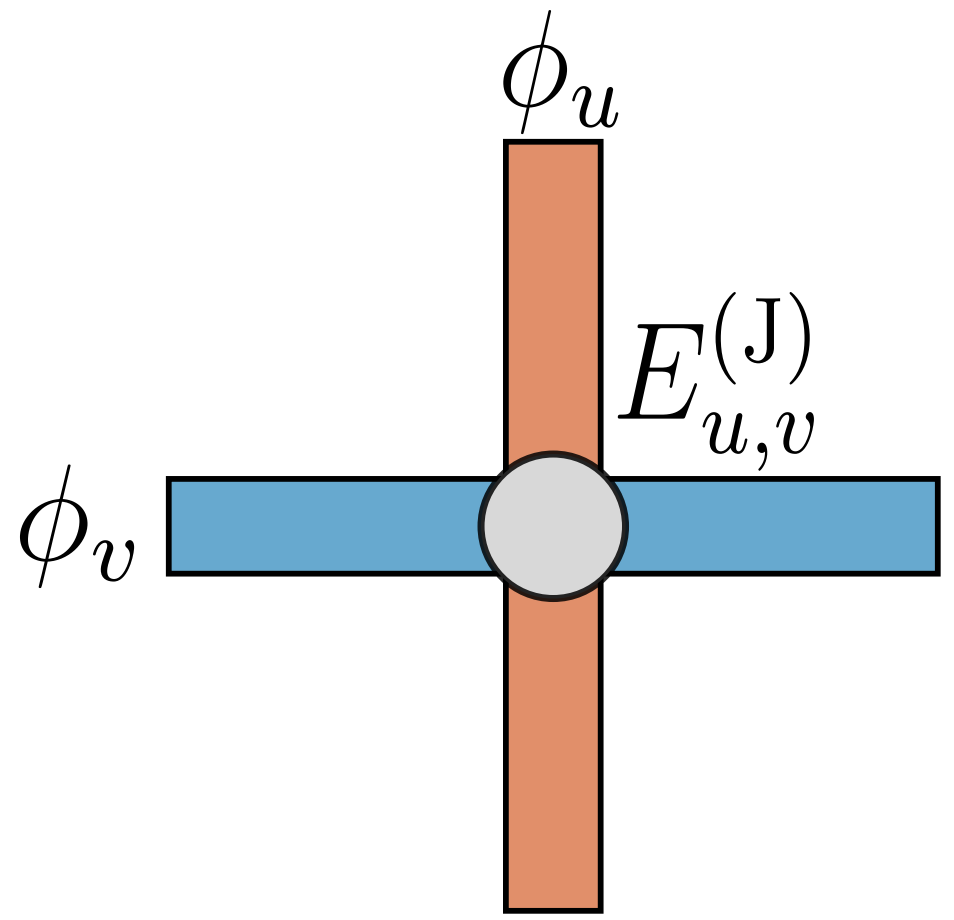
$$\mathscr{F}[\phi] = \frac{1}{2}\int d^3x \, \rho_s \, (\nabla \phi)^2$$

for large stiffness $\rho_s \Rightarrow$ energetic penalty to variations in $\phi(x)$



- Josephson junctions induce energetic interactions of the form

$$H = -E^{(\mathrm{J})}_{u,v} \cos(\phi_u - \phi_v)$$



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Two Steps

1. Triangulate/put a mesh on the manifold M [Cairns (1935) and Whitehead (1939)]

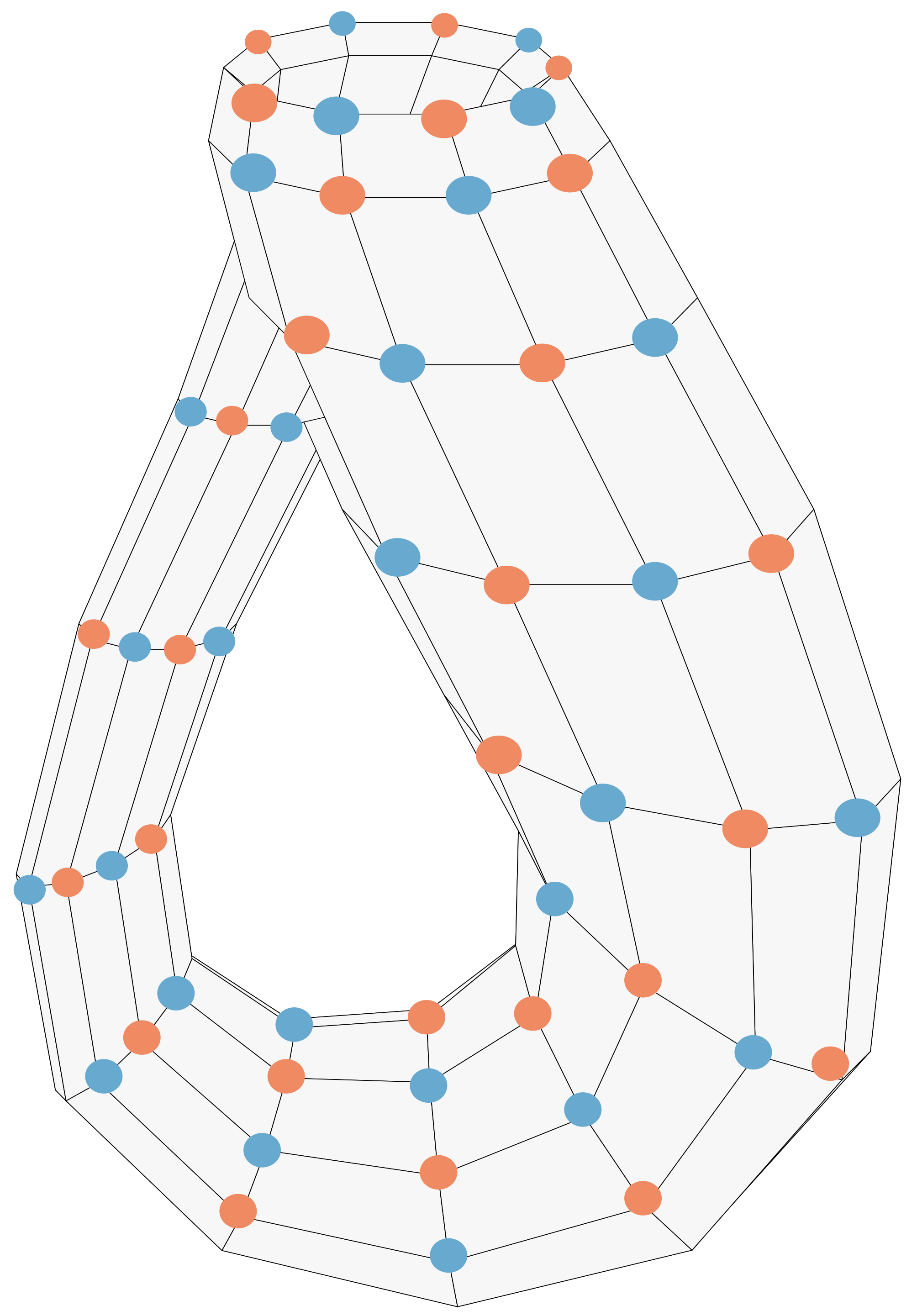
2. Arrange this into a graph structure G = (V,E) on M with

    - Set of nodes (vertices) V

    - Set of links (or edges) E

⇒ The connectivity of the graph can be encoded in a $|V| \times |V|$ adjacency matrix

$$a_{u,v} = 1 \text{ if } \{u, v\} \in E \text{ and } 0 \text{ otherwise.}$$



mesh for Klein bottle

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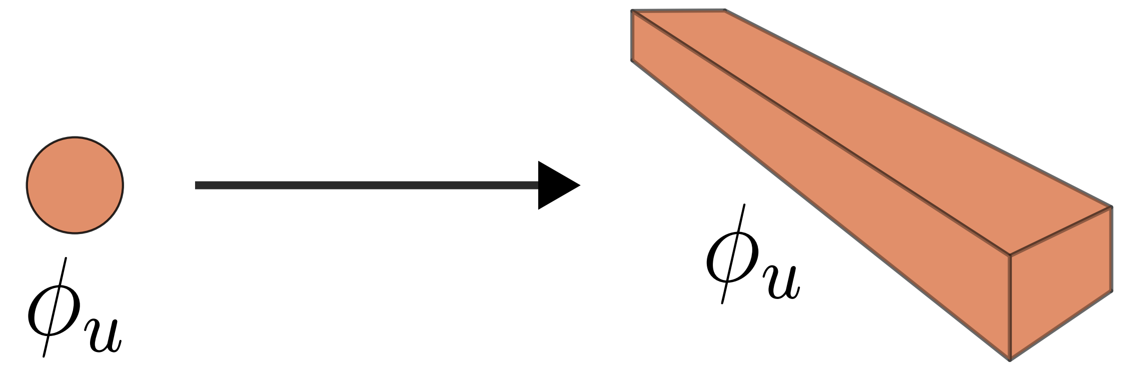
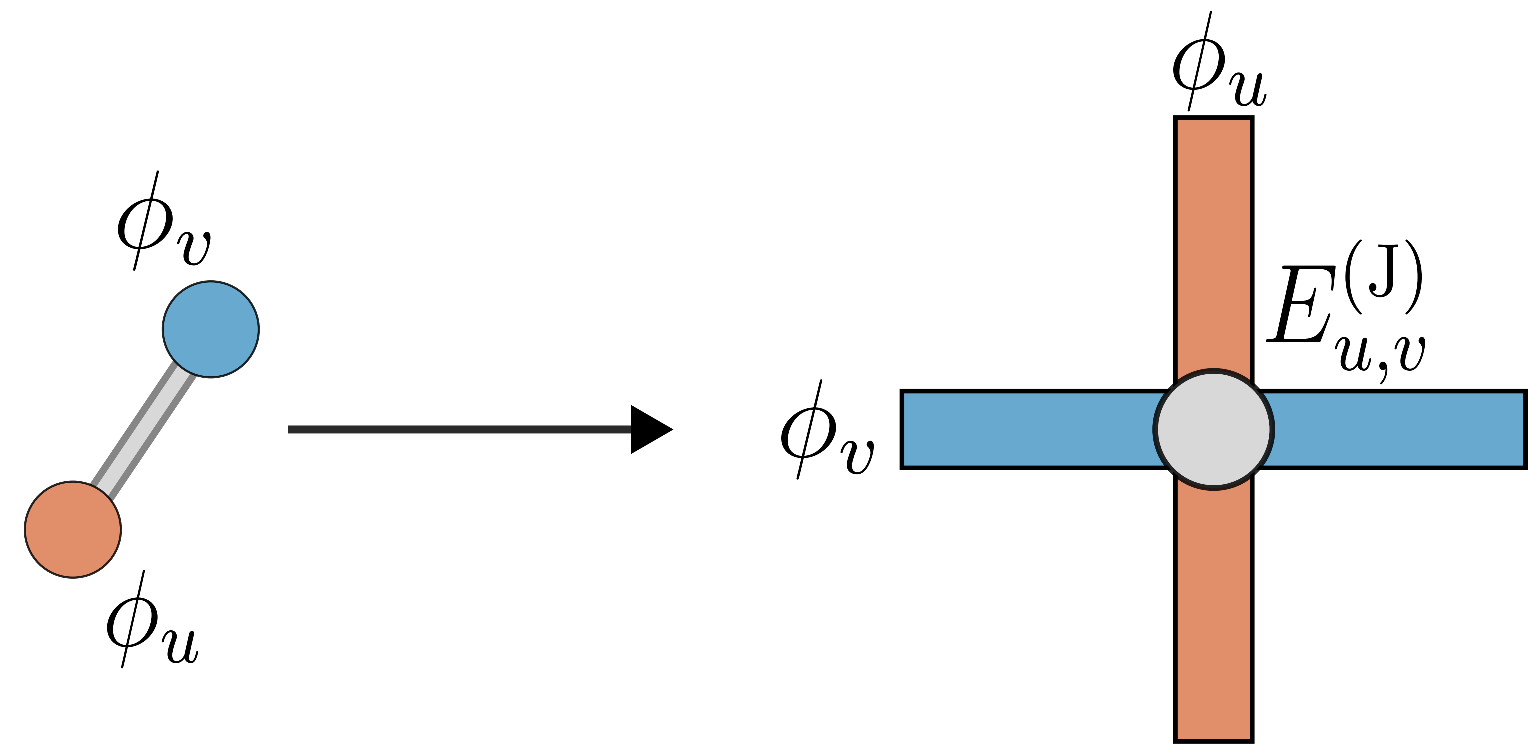
General Framework

Our framework holds for arbitrary simple graphs G

1. To each node $u \in V$, we assign it to a superconducting wire with phase ϕ_u



2. For each edge $\{u, v\} \in E$, we assign it to a Josephson junction with coupling $E_{u,v}^{(J)}$



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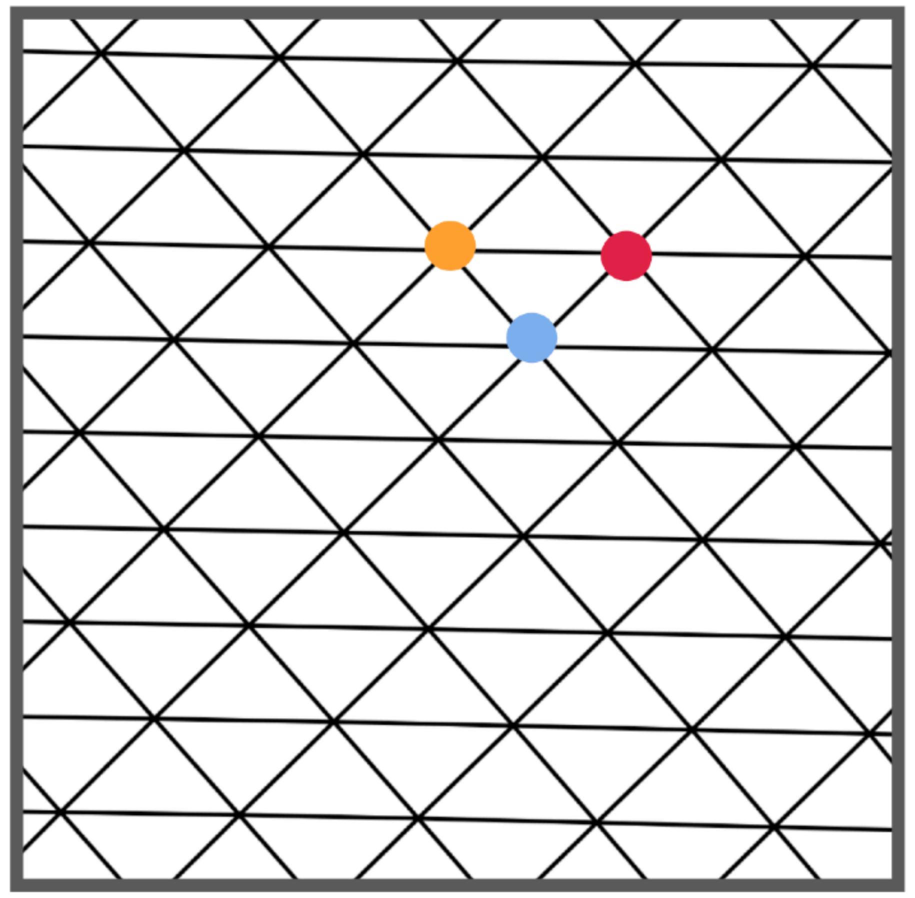
Bipartite Graphs

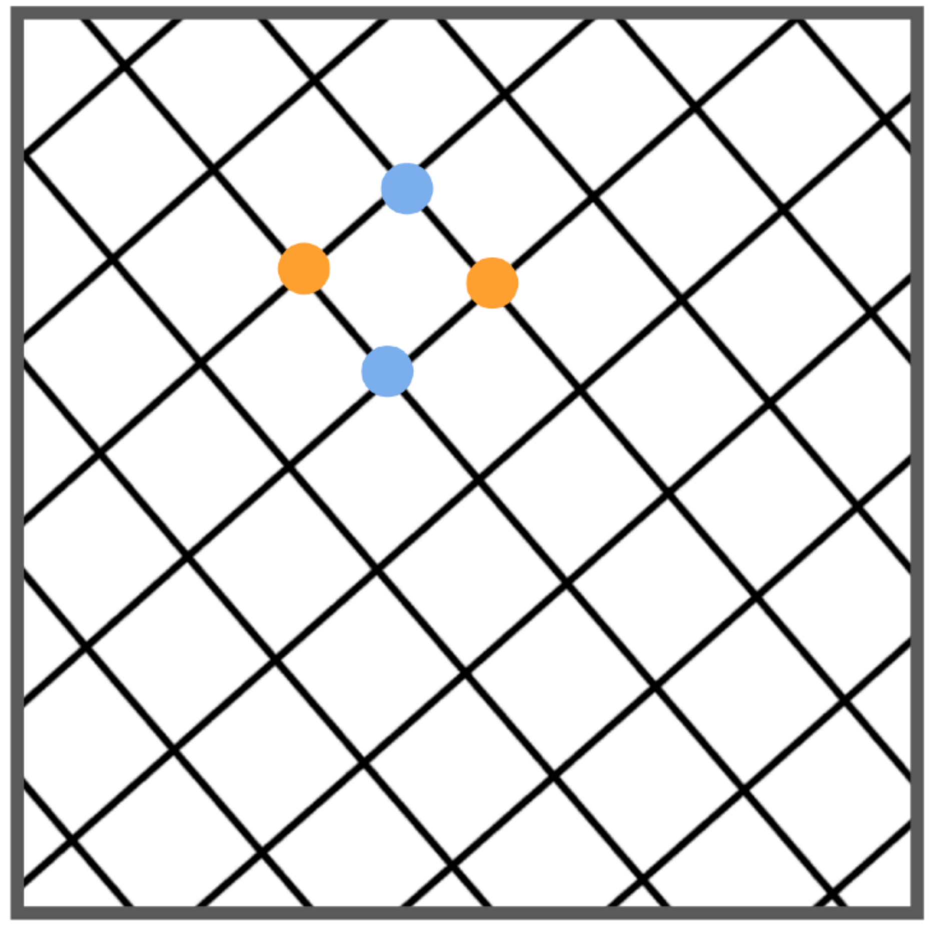


- Although our framework works for arbitrary simple graphs, it simplifies greatly for bipartite ones

- A graph $G = (V, E)$ is said bipartite if $V = V_{\text{orange}} \cup V_{\text{blue}}$

- The adjacency matrix simplifies

$$A = \begin{pmatrix} 0 & B \\ B^\top & 0 \end{pmatrix}, \qquad \text{with } B \text{ the biadjacency matrix}$$

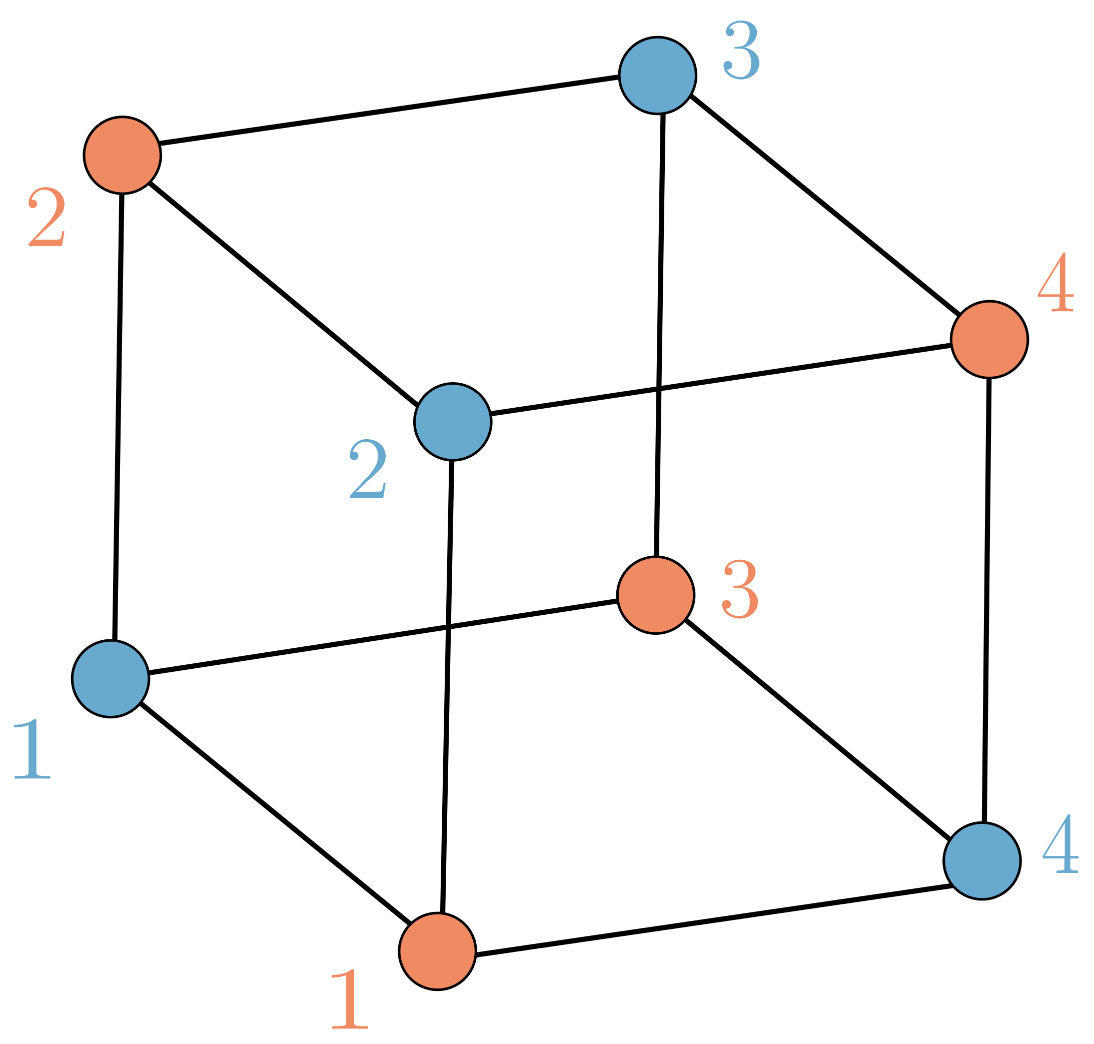


- Then, the SC wire array can be realized through crossing horizontal and vertical wires

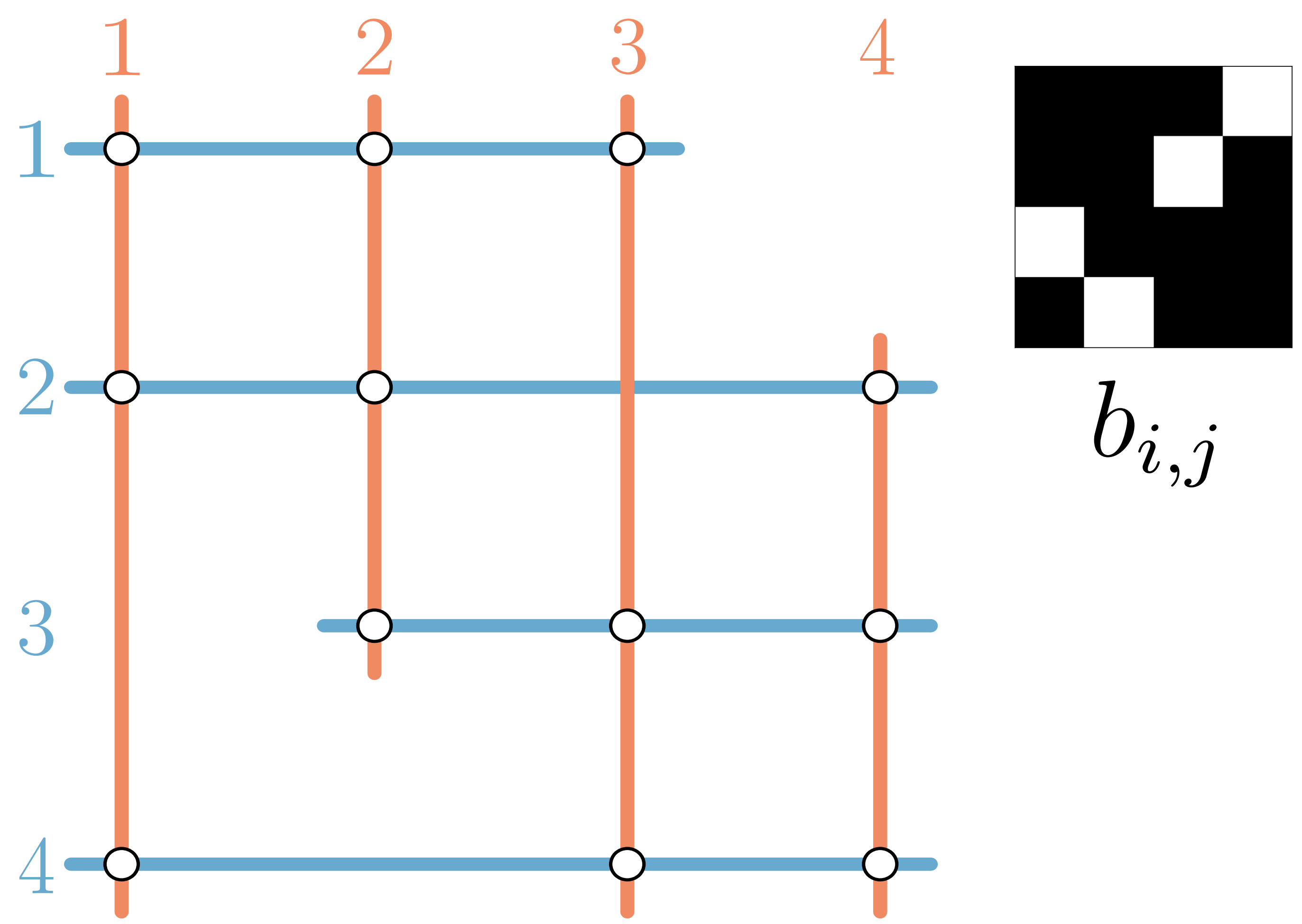
Example: Cube

Under the mapping:

Vertices	⇔	Wires
Edges	⇔	Josephson Junctions
Coordination number	⇔	# of Intersecting wires



⇔



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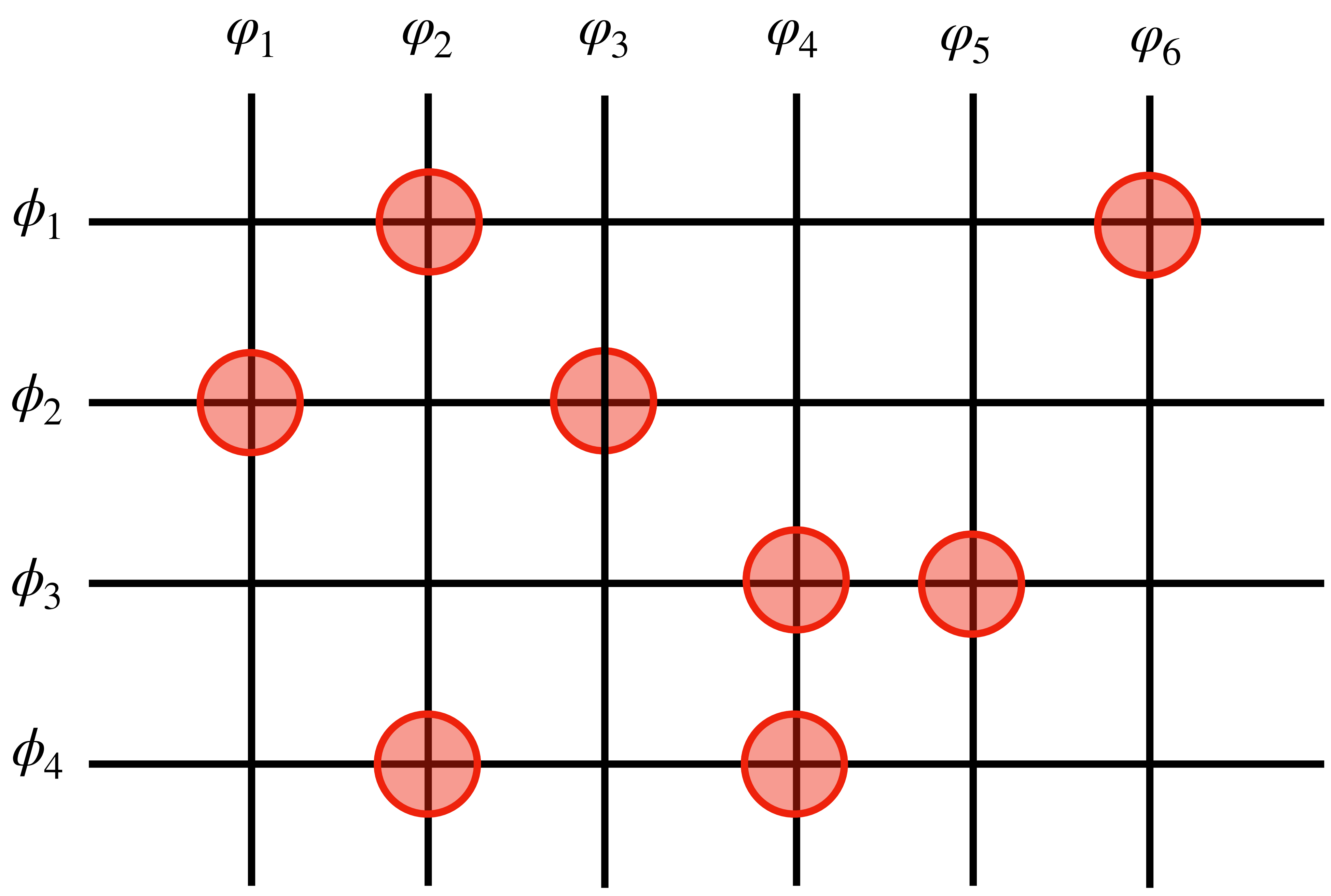
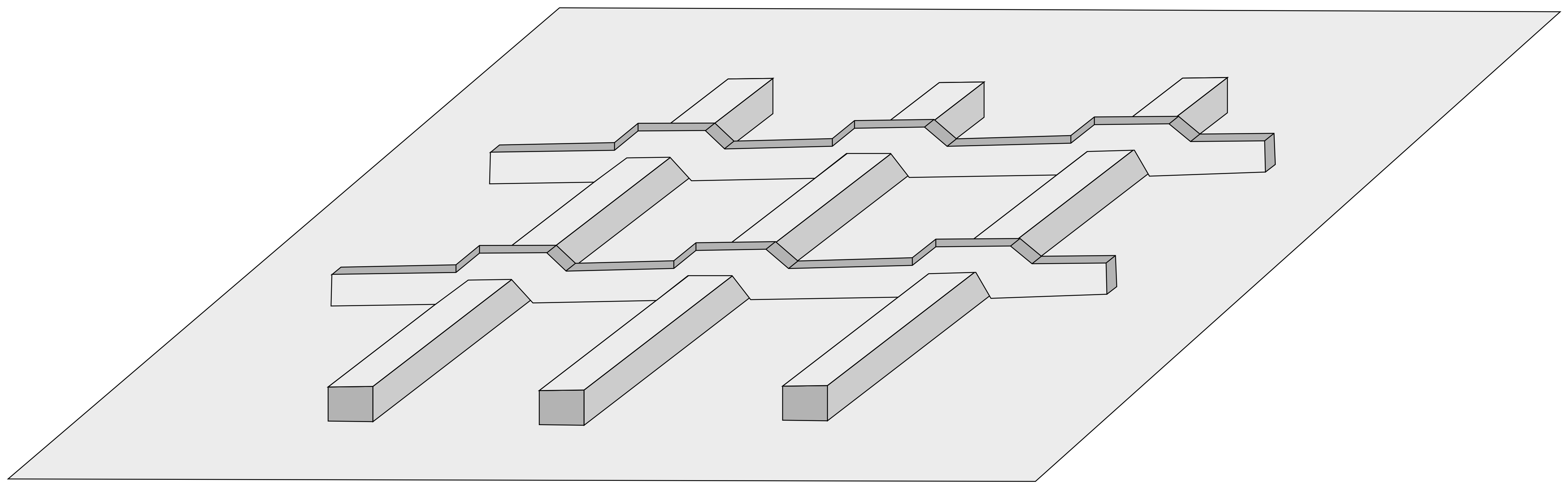
SC wire array as "protoboard"

- The graph $G = (V, E)$ and its biadjacency matrix are the main ingredients in the recipe
- Implementation corresponds to connecting wires together, some version of a "protoboard"



Connectivity matrix $B = (b_{i,j})$

$$H = -J \sum_{i}^{N_1} \sum_{j}^{N_2} b_{i,j} \cos(\phi_i - \varphi_j)$$



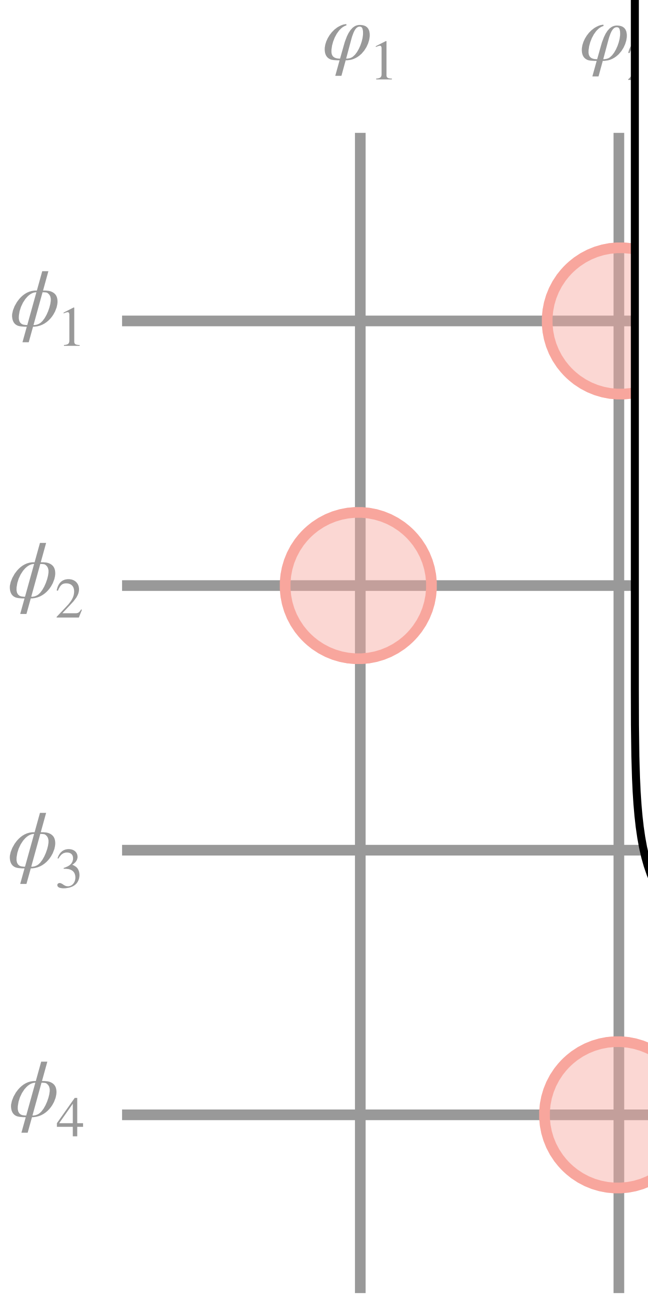
[Sohn et al (1993); Shea & Pinkham (1997)]

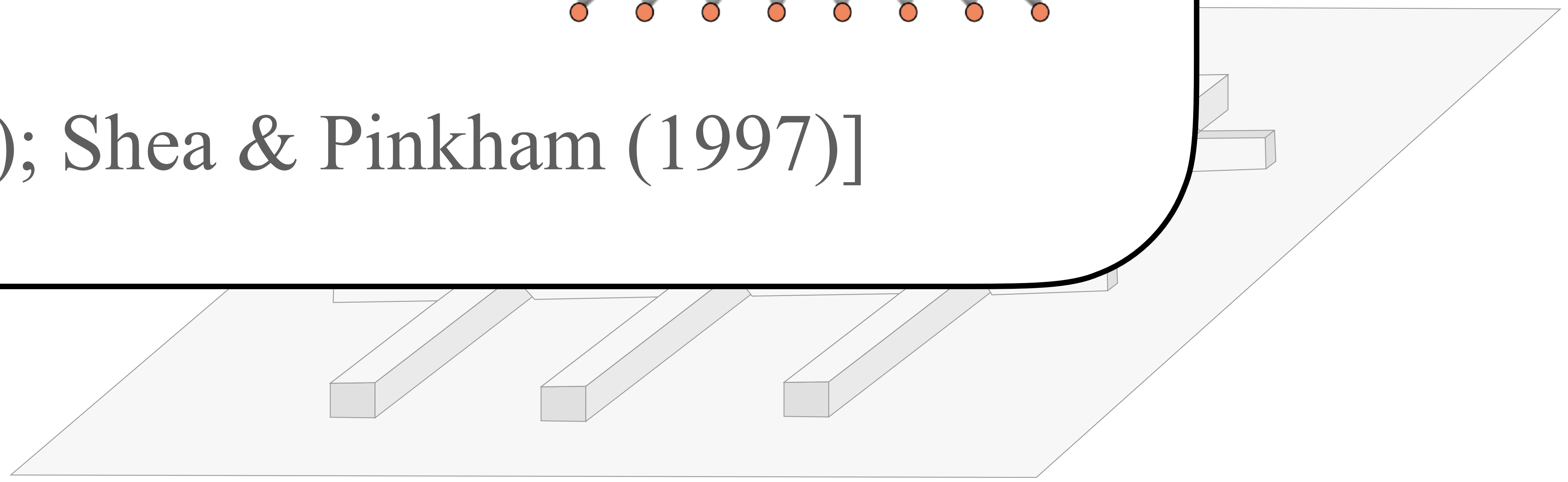
SC wire array as "protoboard"

- The graph G … in the recipe
- Implemen… a "protoboard"

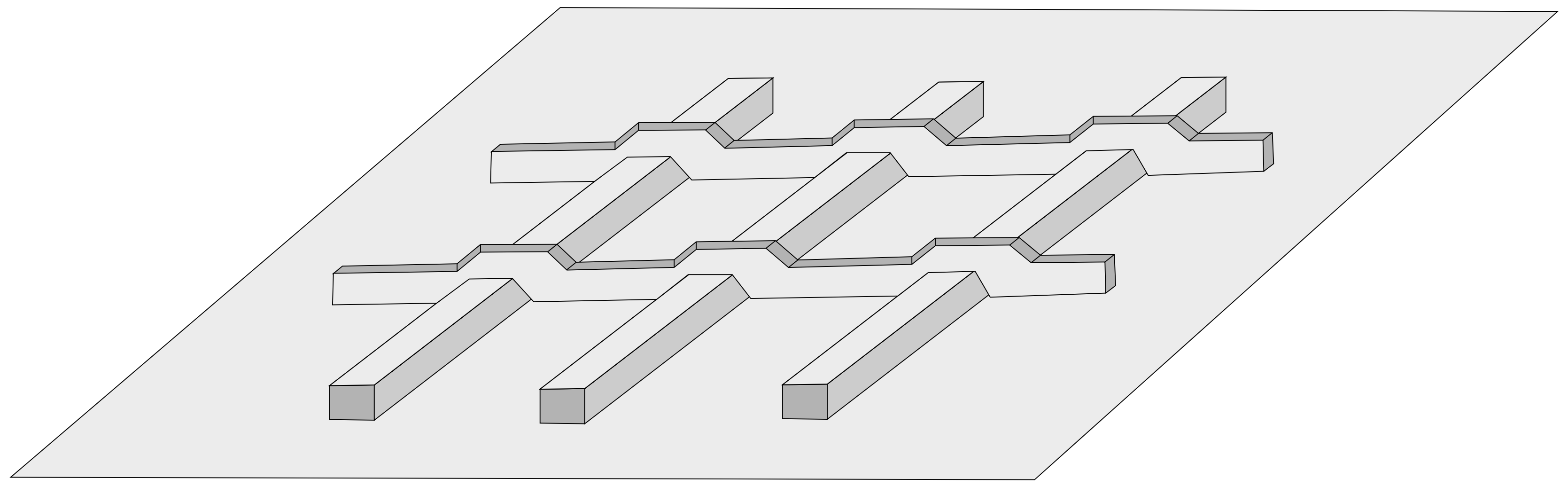
…y matrix $B = (b_{i,j})$

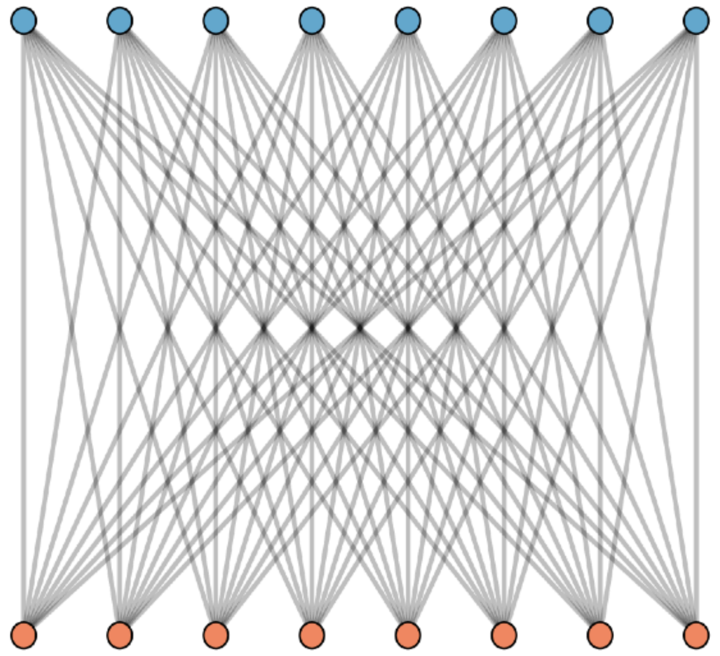
…φ_j)





For $b_{ij} = 1 \, \forall$ pairs $(i, j) \Rightarrow$ Bipartite complete graph $K_{N,N}$





[Sohn et al (1993); Shea & Pinkham (1997)]

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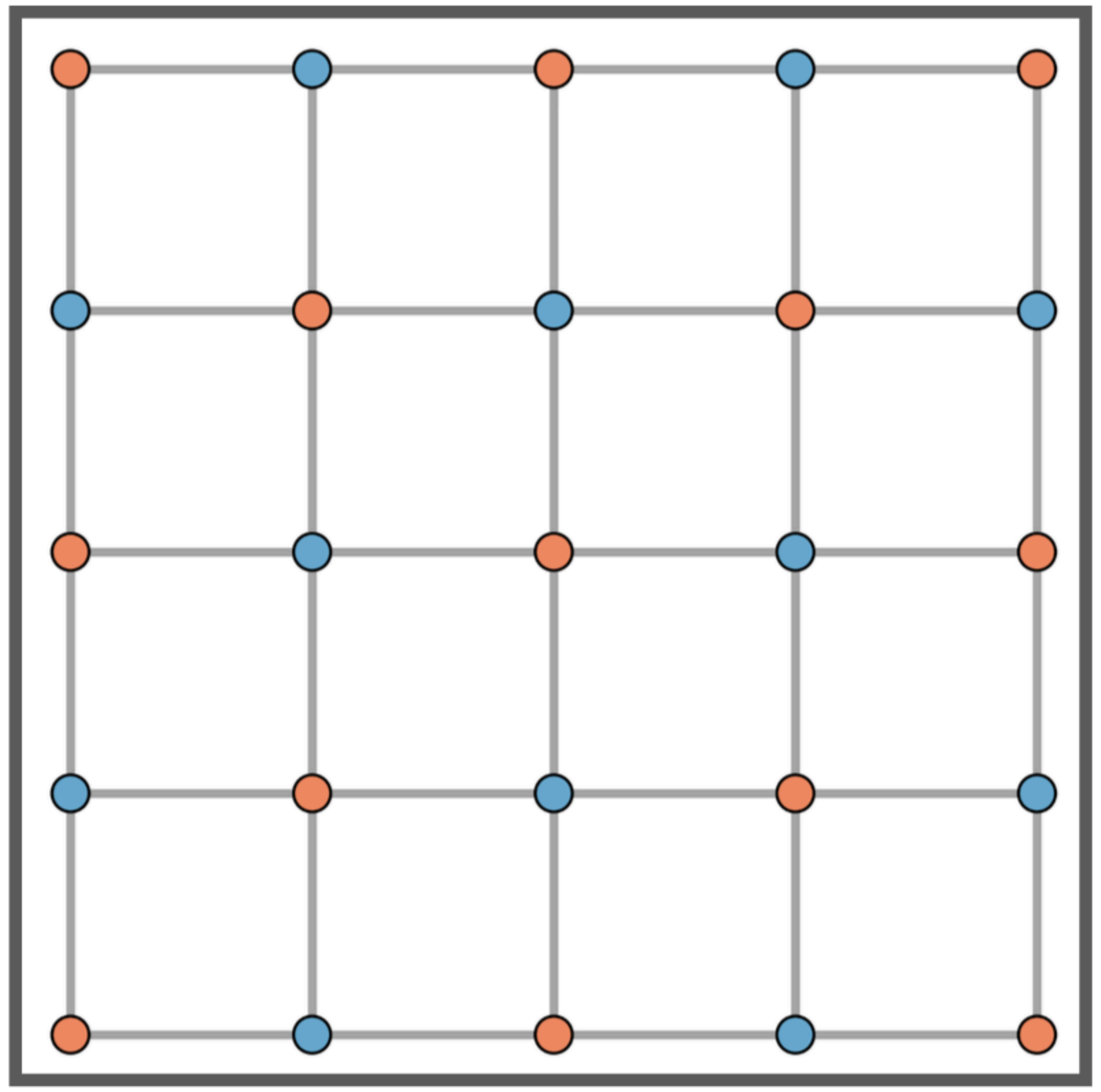
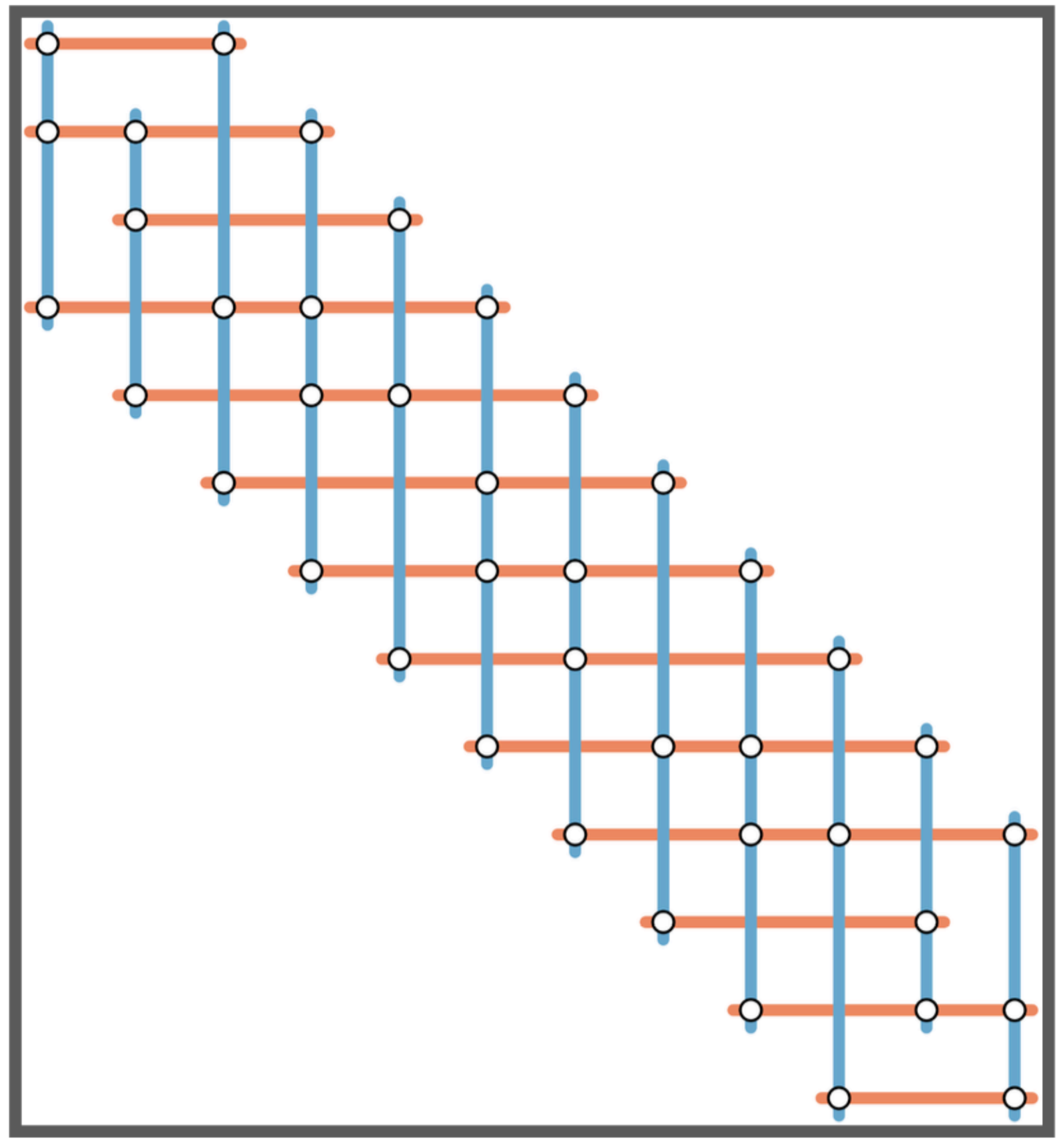
Field Theory @ Square Lattice

$$S_{\mathrm{G}}[\phi] = -J \sum_{\langle i,j \rangle} \cos(\phi_i - \phi_j)$$



$$S_{\mathrm{wire}}[\phi] = -J \sum_{i \in V_{\mathrm{orange}}} \sum_{j \in V_{\mathrm{blue}}} b_{i,j} \cos(\phi_i - \phi_j)$$



$$S_{\mathrm{G}}[\phi] = S_{\mathrm{wire}}[\phi]$$

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Quantum Fluctuations

In general, arrays of Josephson junctions have quantum fluctuations $\left[\hat{\phi}_u, \hat{n}_v\right] = \mathrm{i}\,\delta_{u,v}$ induced by charge operators $\hat{n}_v$. Then $\widehat{H} = \widehat{H}_\mathrm{J} + \widehat{H}_\mathrm{C}$

$$\widehat{H}_\mathrm{J} := -\frac{1}{2}\sum_{u,v} E^{(\mathrm{J})}_{u,v} \cos\left(\hat{\phi}_u - \hat{\phi}_v - A^{(\mathrm{bg})}_{u,v}\right),$$

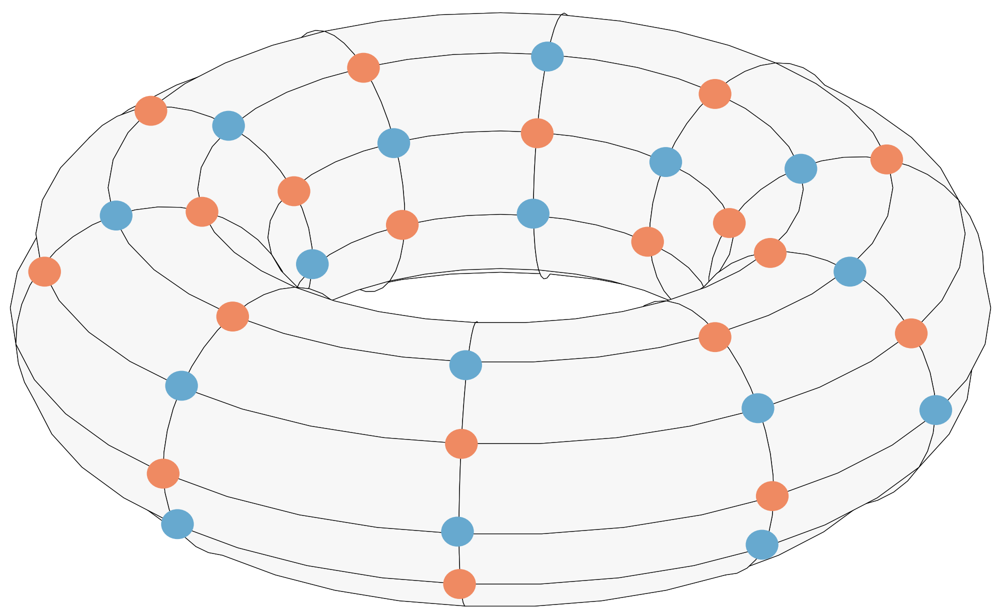
$$\widehat{H}_\mathrm{C} := \frac{1}{2}\sum_{u,v} E^{(\mathrm{C})}_{u,v} \left(\hat{n}_u - n^{(\mathrm{bg})}_u\right)\left(\hat{n}_v - n^{(\mathrm{bg})}_v\right),$$

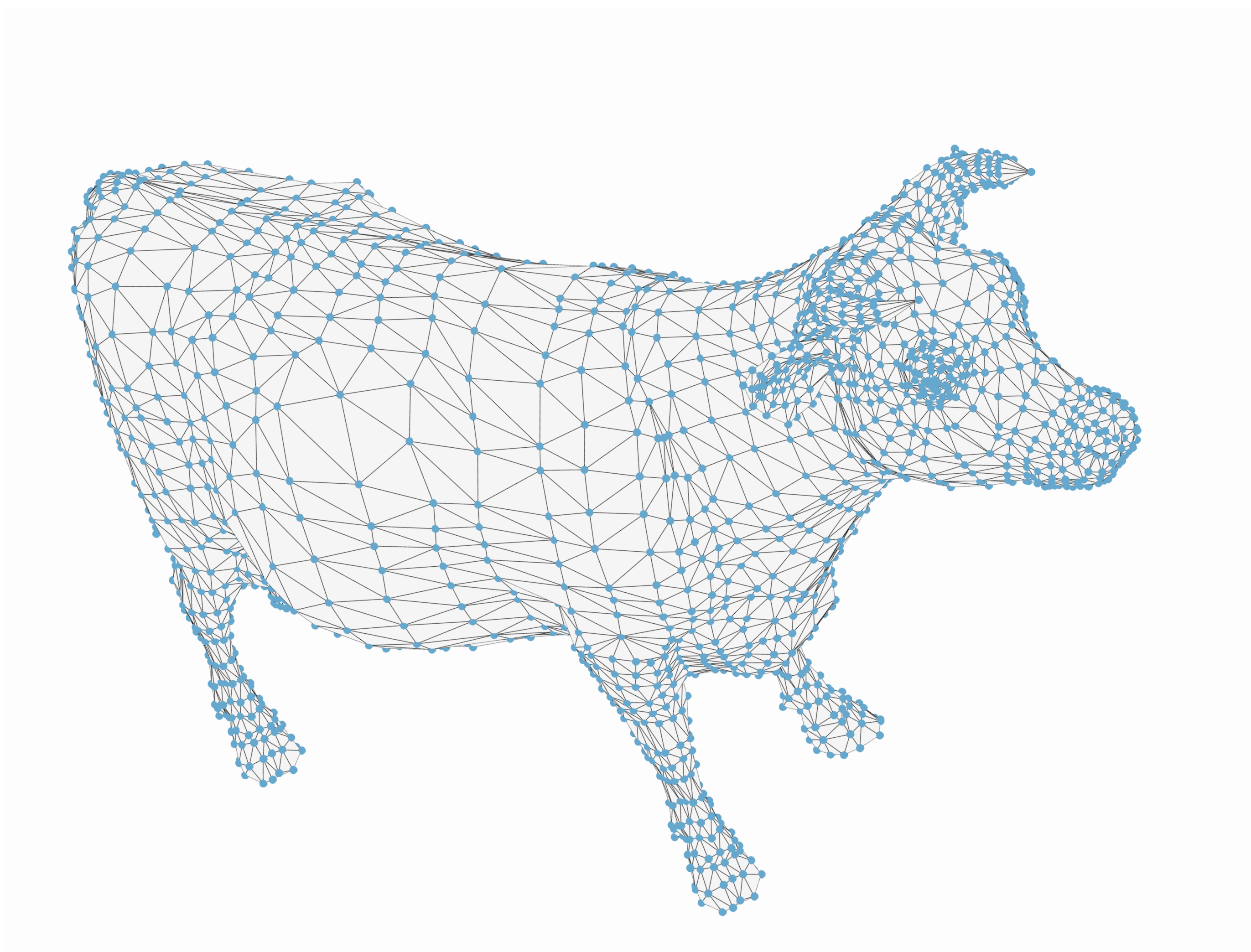
$E^{(\mathrm{J})}_{u,v}$ is the Josephson energy and $E^{(\mathrm{C})}_{u,v}$ is the charging energy between wires u and v, where $E^{(\mathrm{C})}_{u,v}$ is usually set by geometry (wire size).

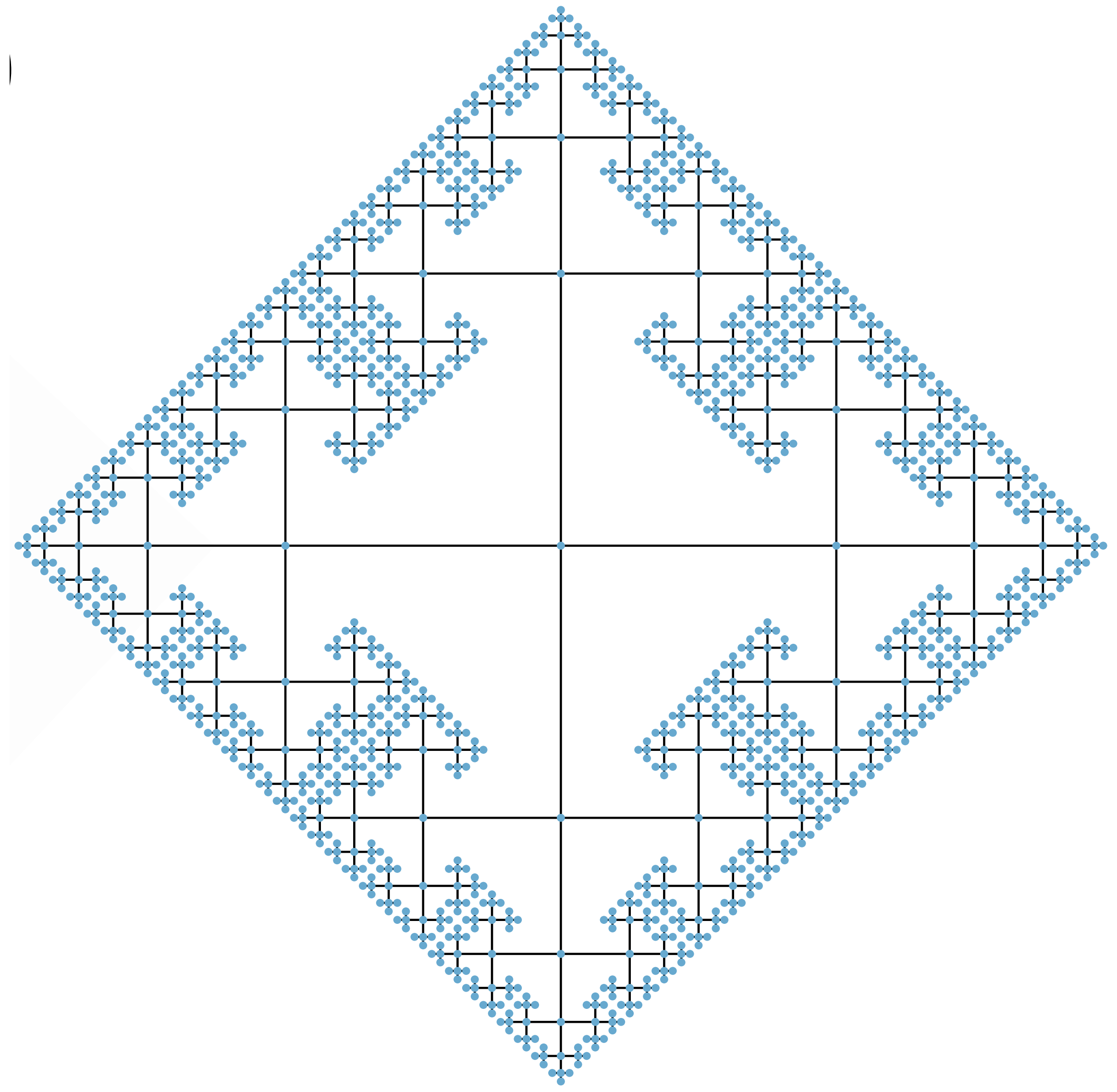
$\Rightarrow$ Classical limit $E^{(\mathrm{C})}_{u,v} \to 0$

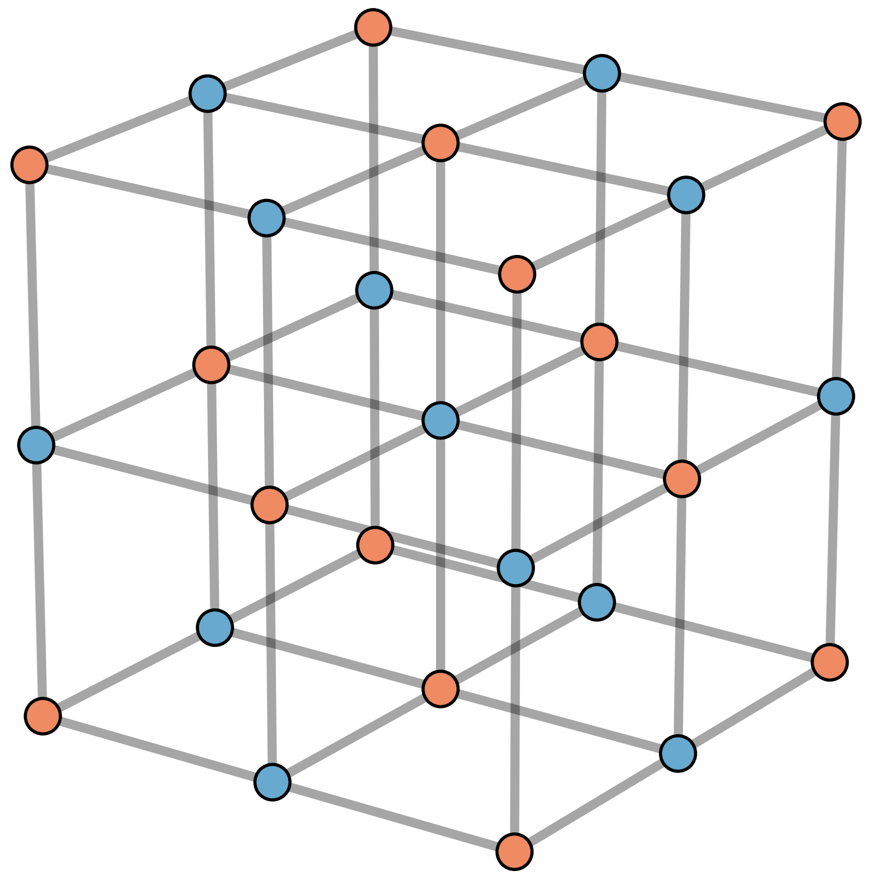
Arbitrary Simple Graphs

- Higher Dimensions
- Curved Spaces
- Cayley graphs for abstract groups
- Etc









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Field Theory @ Anti-de-Sitter spaces

- $\mathrm{AdS}_{\mathrm{d+1}}$ spaces are defined on hyperbolic spaces $\mathrm{d}s^2 = \dfrac{\mathrm{d}z^2 + \mathrm{d}\mathbf{x}^2}{z^2}, \quad z > 0, \quad \mathbf{x} \in \mathbb{R}^d$ with a conformal boundary at $z \to 0$

- Consider the massive scalar field coupled to the above background metric g on $\mathbb{H}^{\mathrm{d+1}}$.

$$S[\phi] = \frac{1}{2} \int_{\mathbb{H}^{d+1}} \mathrm{d}^{d+1}X \sqrt{g} \left(g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi + m^2 \phi^2 \right),$$

- Boundary theory displays power-law decay $\langle \mathcal{O}(\mathbf{x}_1) \mathcal{O}(\mathbf{x}_2) \rangle = \dfrac{c}{|\mathbf{x}_1 - \mathbf{x}_2|^{2\Delta}}$, with

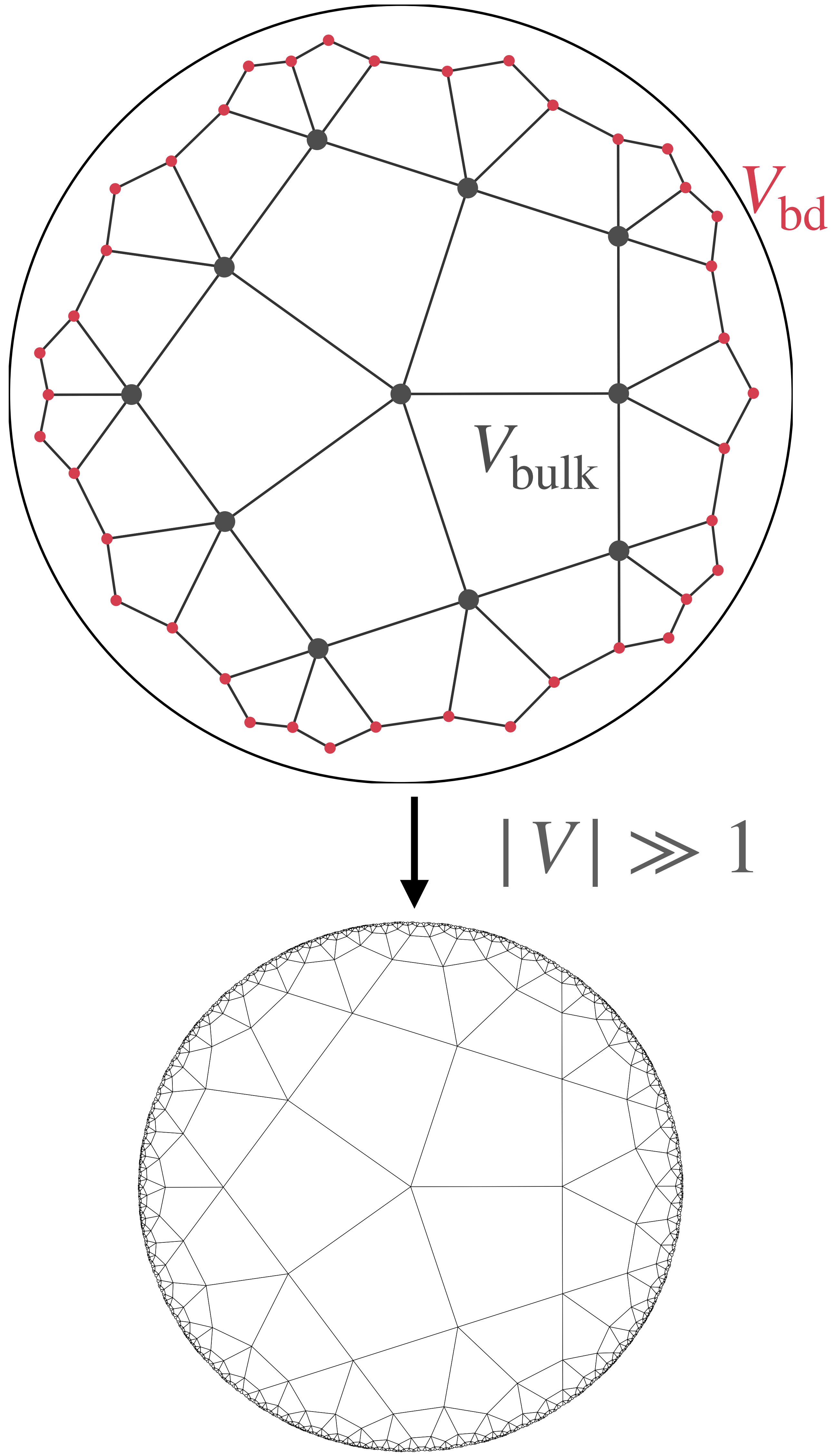
$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{2} + m^2}$$

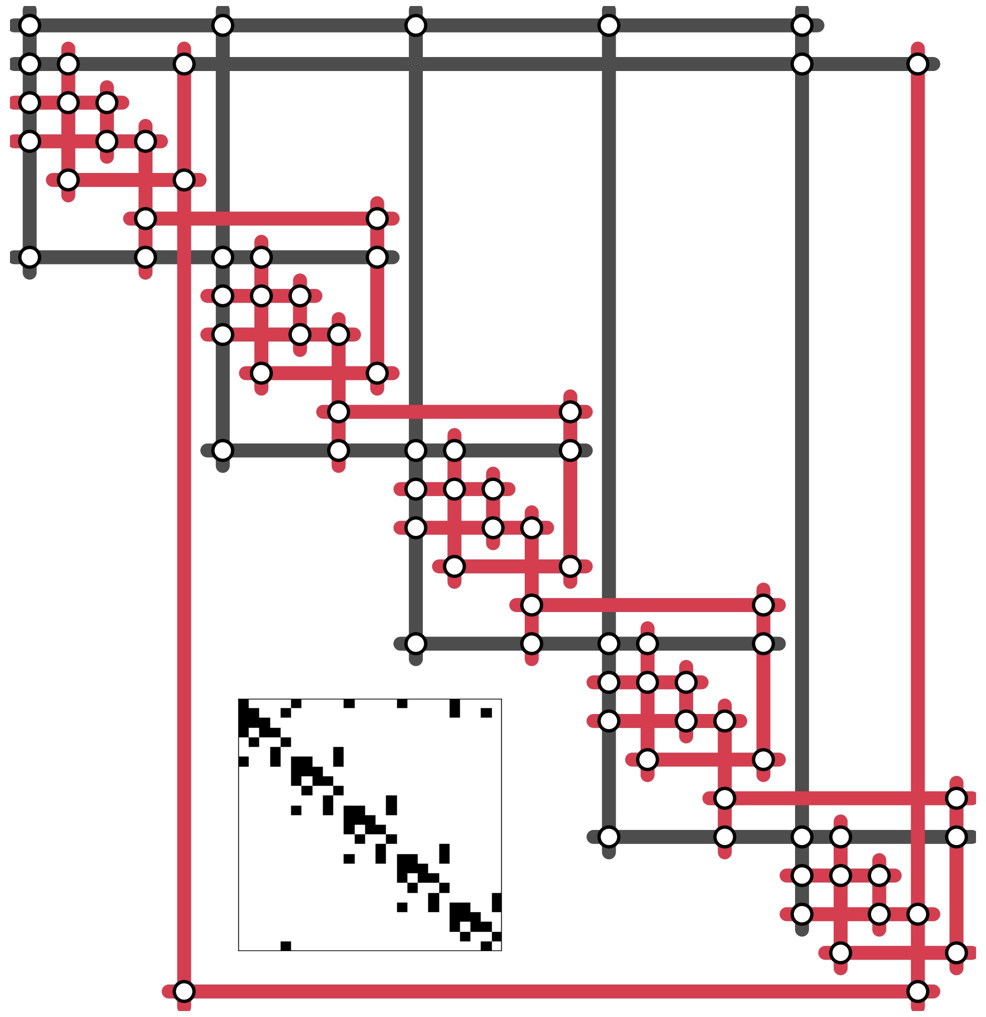
[Witten (1998)]

Setup

- Let us consider the $\{4,5\}$ discretization of $\mathbb{H}_2$ place in Poincare-disk coordinates $G = (V, E)$, with $V = V_{\text{bulk}} \cup V_{\text{bd}}$

- To account for mass terms, we add a “big” superconductor ϕ_∞ that biases all other phases



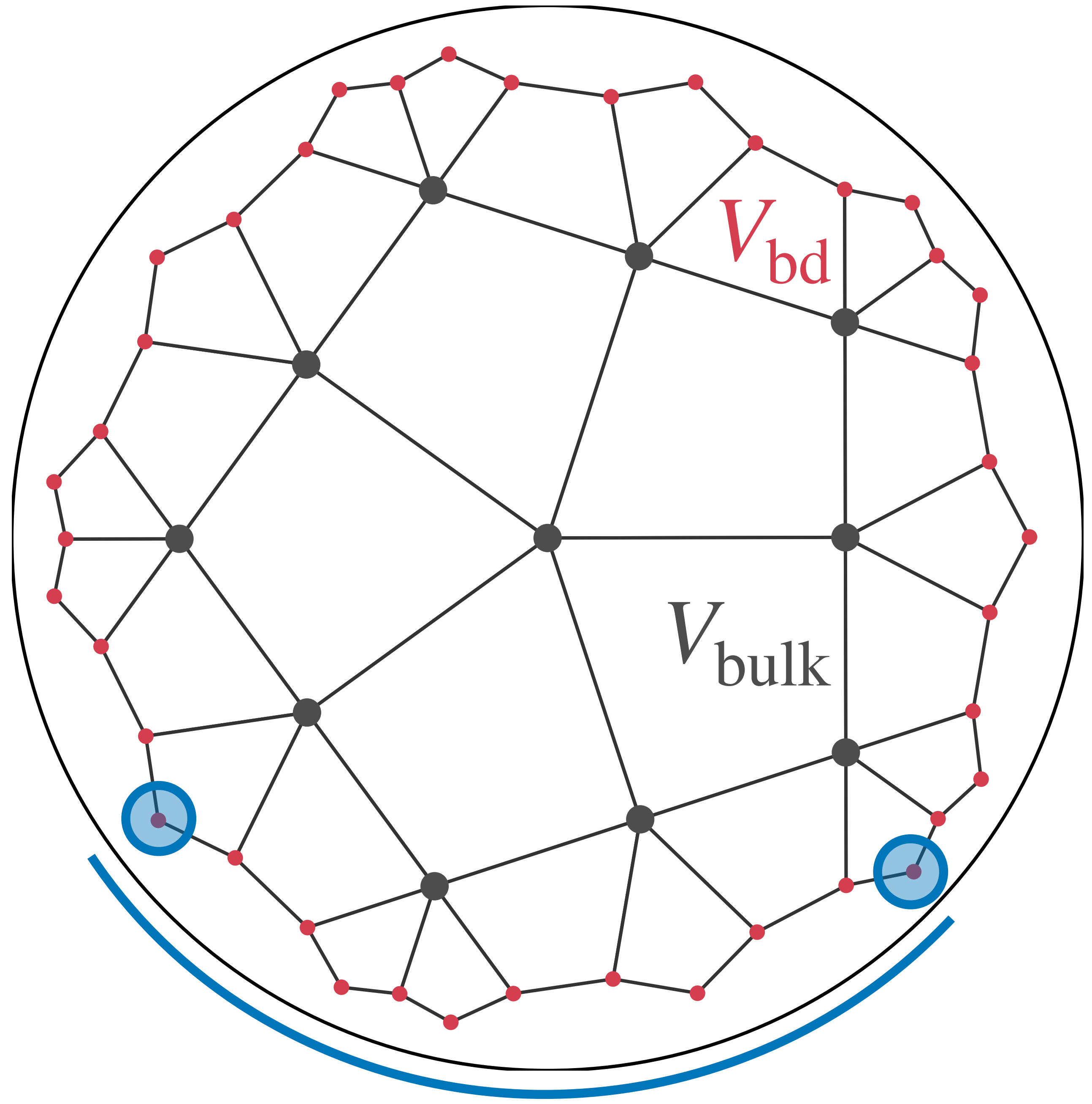


$$H_G = \frac{1}{2} \sum_{u,v \in V} a_{u,v}\, J\, [1 - \cos(\phi_u - \phi_v)] + \sum_{u \in V} m^2\, [1 - \cos(\phi_u - \phi_\infty)] + \sum_{u \in V_{\text{bd}}} M^2\, [1 - \cos(\phi_u - \phi_\infty)]\,,$$

Boundary-boundary propagators

- Let us consider boundary points and compute edge propagators

$$G^{(0)}_{\text{edge}}(r) \equiv \frac{\sum_{u,v\in V_{\text{bd}}} \langle \phi_u \phi_v \rangle \, \delta_{r,d(u,v)}}{\sum_{u,v\in V_{\text{bd}}} \delta_{r,d(u,v)}} \qquad \text{for } u, v \in V_{\text{bd}}$$



Where $d(u, v)$ is the distance between u and v along the boundary

From conformal invariance, we expect

$$G^{(0)}_{\text{edge}}(r) \sim \left[\frac{1}{2\,\sin^2\left(\pi\, r/r_{\max}\right)}\right]^{\Delta} \approx r^{-2\Delta}$$

r: boundary distance

$r_{\max}$: perimeter of boundary

$2\sin^2(\pi r/r_{\max})$: conformal chordal distance

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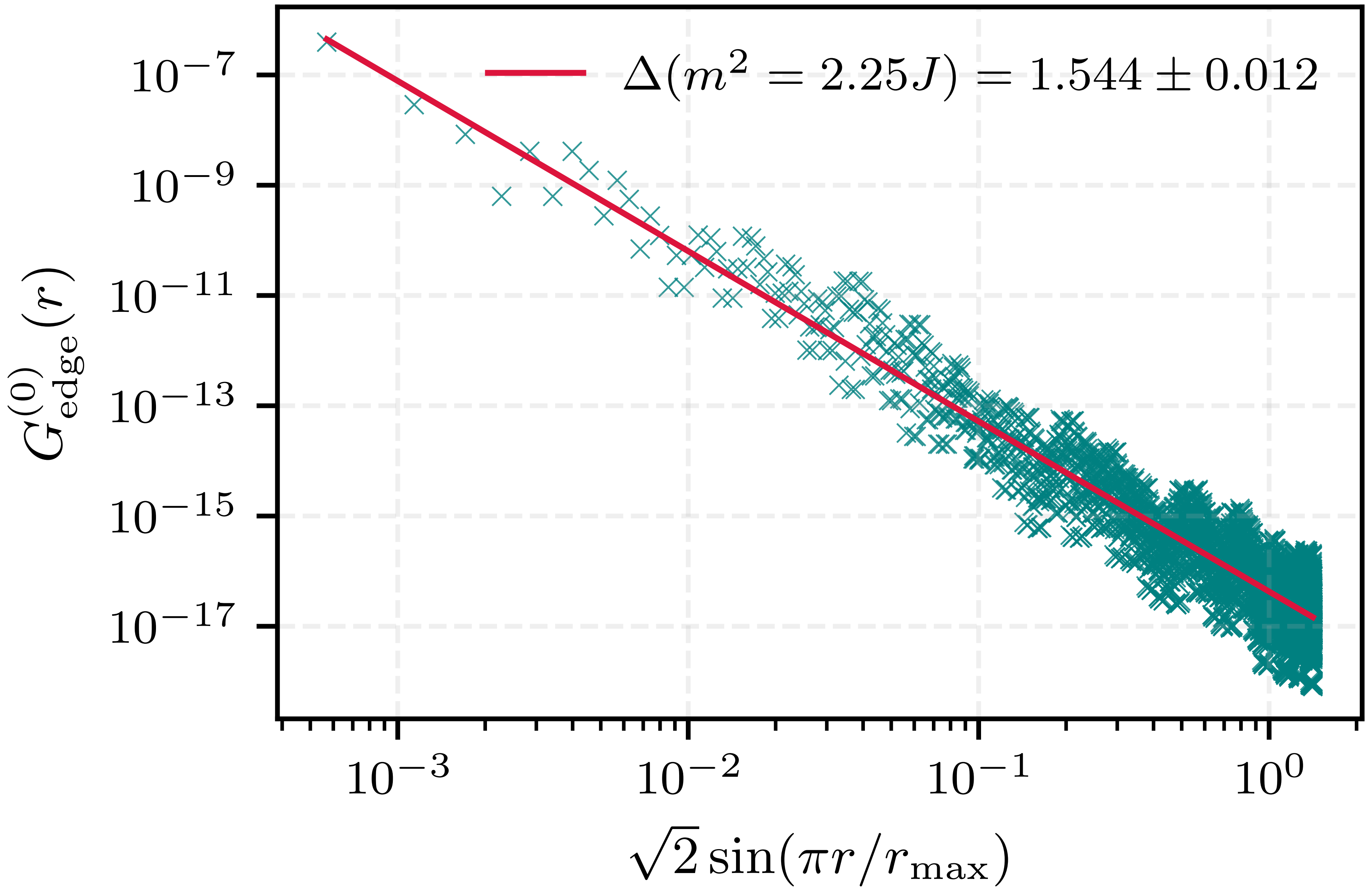
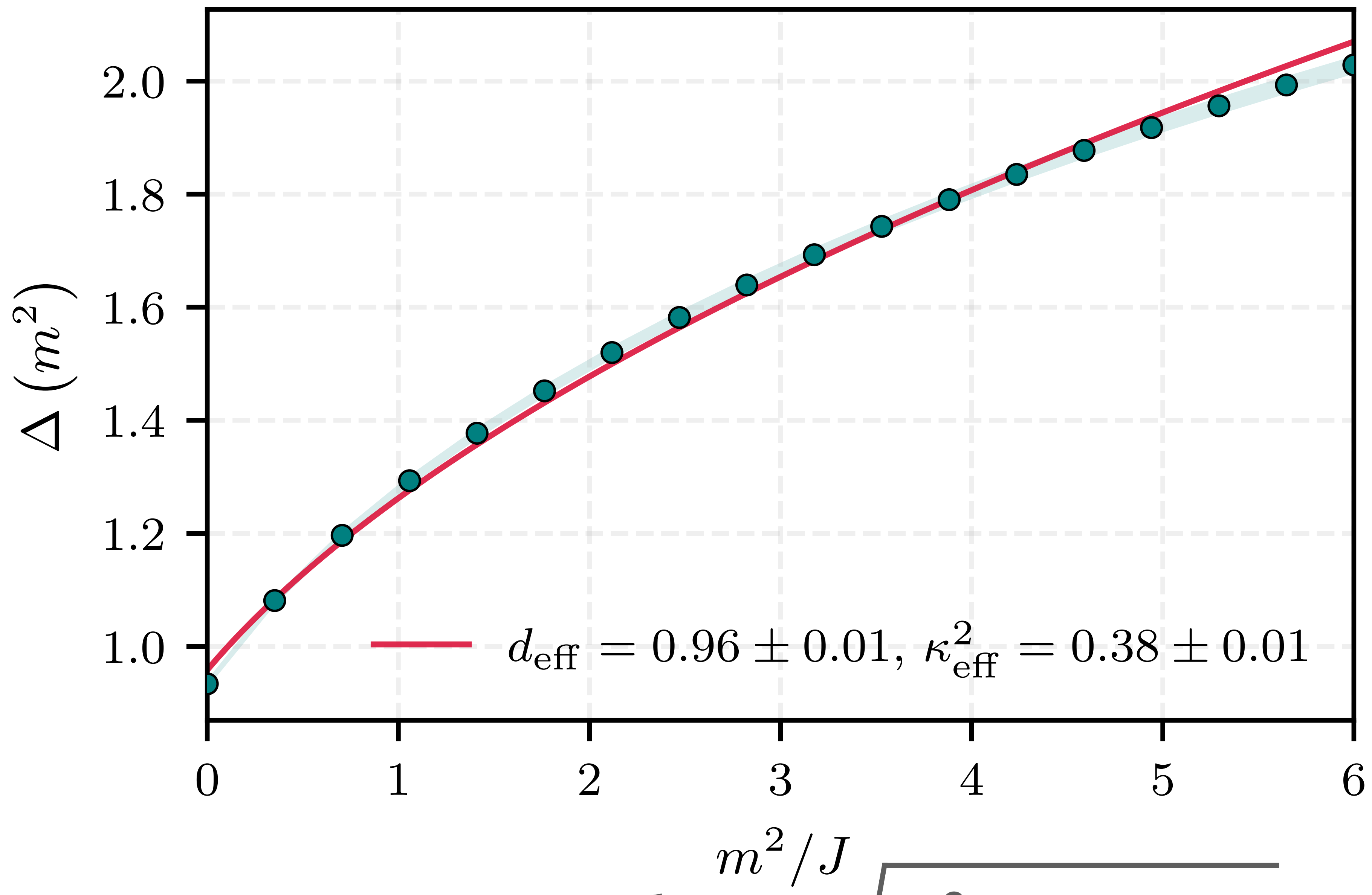
Numerical Results

- We observe a straight line, indicating the power-law behavior with coefficient $2\Delta(m)$





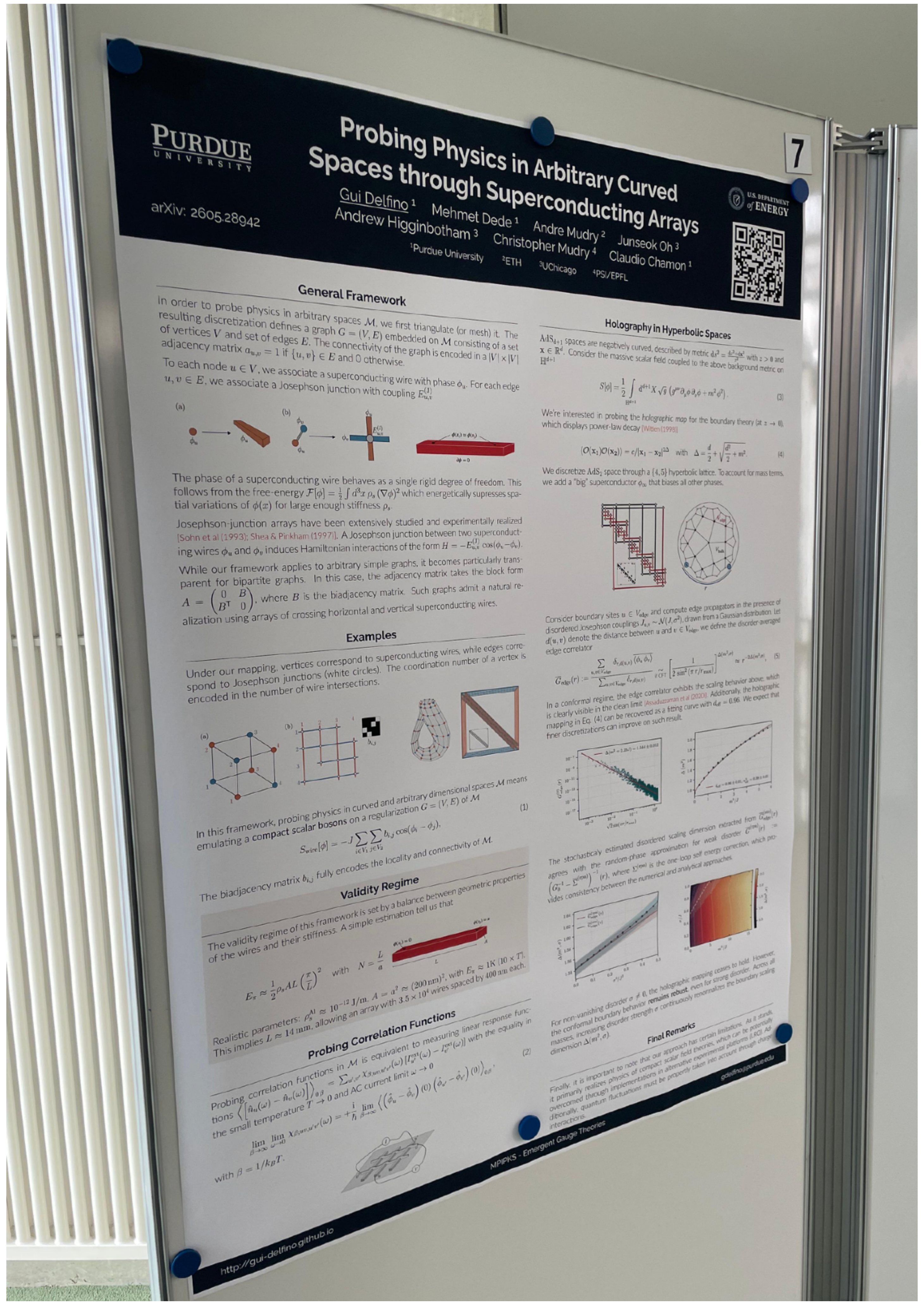
$$\Delta(m) = \frac{d_{eff}}{2} + \sqrt{\frac{d_{\mathrm{eff}}^2}{4} + \kappa_{\mathrm{eff}}^2 m^2}$$

⇒ How stable are the results against imperfections?

[Assaduzzaman et al (2020)]

See poster #7 upstairs for more!

- Estimation of validity of of our framework;
- Robust of holographic mapping against disorder & benchmark with replica theory;
- General experimental considerations
- How to probe correlation functions experimentally?



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Thank you!

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